THE

COMPLETE MEASURER:

OR, THE

Whole Art of Measuring.

IN TWO PARTS.

PART I.

Teaching *Decimal Arithmetic*, with the *Extraction* of the *Square* and *Cube-Roots*.

Alſo the *Multiplication* of *Feet* and *Inches*, commonly called *Croſs-Multiplication*.

PART II.

Teaching to meaſure all Sorts of Superficies and Solids, by *Decimals*, by *Croſs-Multiplication*, and by *Scale* and *Compaſſes*: Alſo, the Works of ſeveral Artificers relating to Building, and the meaſuring of Board and Timber, ſhewing the common Errors, and ſome Practical Queſtions.

Thirteenth Edition.

To which is added an APPENDIX. 1. Of *Gaging*. 2. Of *Land-Meaſuring*.

VERY USEFUL FOR ALL TRADESMEN, ESPECIALLY CARPENTERS, BRICKLAYERS, PLASTERERS, PAINTERS, JOINERS, GLAZIERS, MASONS, &c.

BY WILLIAM HAWNEY, PHILOMATH.

Recommended by the REV. DR. JOHN HARRIS, F. R. S.

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PREFACE.

HAVING peruſed ſeveral books concerning the menſuration of ſuperficies and ſolids, and the works of Artificers relating to building; but not finding any one book ſo perfect, as to give any tolerable ſatisfaction to a learner; and I having practiſed and taught meaſuring for ſeveral years, and thereby gained experience and knowledge in that art, having learned ſome things from one author, and ſome things from another, I began to think of digeſting my thoughts into ſome ſuch method as might give a learner full ſatisfaction, without being at the charge of buying ſo many books; and being importuned thereunto by ſome friends, I fell to work, and at laſt brought them to that perfection you here find in the following WORK.

1. As to the DECIMAL ARITHMETIC, I have been as brief as the matter would well bear to make it plain.

2. As to the multiplying of ſeet and inches, commonly called CROSS MULTIPLICATION, my method differs from that

which is uſually taught in other authors, as being (I think) much ſhorter and plainer.

3. In meaſuring of ſuperficies and ſolids, I have given the demonſtration of the rules, which I thought might be very acceptable to the ingenious; for indeed, I always look upon the writing of a rule without a demonſtration, (in any part of the mathematicks) to be but lame and defective; and for want of knowing the reaſon of the rule, a learner may commit great errors: beſides, when a learner knows the reaſon of the rules, he may retain them better in his memory. The rule for meaſuring a priſmoid and cylindroid, I had out of Mr. *Everard's Art of Gaging;* but the reaſon he does not ſhew, neither have I found it in any other author; but that the method is true, I have endeavoured to make plain.

The demonſtrations of the rules for finding the area of an ellipſis and parabola; alſo the demonſtration of the rules for finding the ſolid content of the fruſtum of a cone and pyramid, the ſolidity of a globe, of a ſpheroid, a parabolic conoid, and of a parabolic ſpindle, and their fruſtums, I had from the ingenious Mr. *Ward's Young Mathematician's Guide;* where the curious and ingenious reader may ſee many other demonſtrations algebraically performed: I have alſo demonſtrated the rule for finding the ſolidity of a globe, out of *Pardie's Elements of Geometry*, (Book Vth. Art. the 33d.) publiſhed in *Engliſh*, with many additions, by the Rev. Dr. *Harris*, F. R. S. and the ſame is alſo done out of *Sturmius's Matheſis Ennucleata*; ſo that the ingenious reader may uſe which of thoſe ways he likes beſt.

The Scale ſuppoſed to be uſed in all the operations is the line of numbers, commonly called *Gunter's Line*, which is upon the ordinary two feet or eighteen inch rules, commonly uſed by the carpenters, maſons, &c. becauſe I thought it needleſs, as well as impertinent, to write the uſe of ſliding-rules, or any other particular ſcales they being ſufficiently treated of by ſeveral authors, *viz.* by the above-named Mr. *Everard* in his *Art of Gaging*, above-mentioned, where you have the uſe of a ſliding-rule in arithmetic, geometry, in meaſuring of ſuperficies and ſolids, gaging, &c. Likewiſe Mr. *Hunt* has wrote largely of the uſe of his ſliding rule, in arithmetic, geometry, trigonometry, gaging, dialling, &c. There are ſeveral others who have explained the uſe of their own rules; ſo that the more curious readers may find full ſatiſfaction in thoſe authors.

One thing I have omitted in the book, which I think may not be very improperly inſerted in this place; that is, how to find a number upon the line. If the number you will find conſiſts only of units, then the figures upon the line repreſent the number ſought: Thus, if the number be 1, 2, 3, &c. then 1, 2, 3, &c. upon the line, repreſent the number ſought; but if the number conſiſts of two figures, that is, of units and tens, then the figure upon the rule ſtands for tens, and the larger diviſions ſtand for units: thus, if 34 were to be found upon the line, the figure 3 upon the line is 30, and 4 of the large diviſions (counted forward) is the point repreſenting 34; and if 340 were to be found, it will be at the ſame point upon the line; and if 304 were to be found, then the 3 upon the line is 300, and 4 of the ſmaller diviſions (counted forward) is the point repreſenting 304: If the number conſiſts of four places, or thouſands, then the figure upon the line ſtands for thouſands, and the larger diviſions are hundreds, the leſſer diviſions are tens, and the tenth parts of thoſe leſſer diviſions are units; thus, if 2735 were to be found, then the 2 is 2000, and the 7 larger diviſions (counted forward) is 700 more, and 3 of the leſſer diviſions is 30 more, and half of one of the leſſer diviſions is 5 more, which is the point repreſenting 2735. You muſt remember, that between each figure upon the line there are ten parts, which I call the larger diviſions, and each of thoſe larger diviſions are ſubdivided (or ſuppoſed ſo to be) into ten other parts, which I call the ſmaller diviſions, and each of thoſe parts ſuppoſed to be ſubdivided again into ten other parts, *&c.* You muſt alſo remember, that if 1, in the middle of the line, ſtands only for 1, then 1 at the upper end will be 10, and one at the lower end will only be $\frac{1}{10}$; but if 1 at the lower end ſignifies 1, then 1 in the middle ſtands for 10, and 1 at the upper end is 100, *&c.*

There is one thing more which I would have my reader to underſtand, and that is, how to find all ſuch proportional numbers made uſe of in the proportions about a circle, and of a cylinder, and in other places; which thing may be of good uſe, to know how to correct a number which may happen to be falſe printed, or to enlarge any number to more decimal places, for more exactneſs; for though I have mentioned what ſuch numbers are, yet I have not ſhewn how to find them, which a learner may be a little at a nonplus to do, though they are eaſily found by the rules there laid down.

I ſhall therefore give two or three examples, in this place, of finding ſuch numbers, which may enable my reader to find out the reſt.

And, firſt, let it be required to find the area of a circle, whoſe diameter is an unit.

By the proportion of *Van Culen*, if the diameter be 1, the circumference will be 3.1415926, *&c.* whereof 3.1416 is ſufficient in moſt caſes. Then the rule teaches to multiply half the circumference by half the diameter, and the product is the area, that is, multiply 1.570 by .5, (*viz.* half 3.1416 by half 1) and the product .7854. which is the area of the circle, whoſe diameter, is 1.

Again, if the area be required, when the circumference is 1; firſt, find what the diameter will be, thus, as 3.1416 : to 1 : : ſo is 1 : to .318309, which is the diameter when the circumference is 1. Then multiply half .318309 by half 1, that is, .159154 by 5, and the product is .079577, which is the area of a circle whoſe circumference is 1.

If the area be given to find the ſide of the ſquare equal, you need but extract the ſquare root of the area given, and it is done: ſo the ſquare root of .7854 is .8862, which is the ſide of a ſquare equal when the diameter is 1. And if you extract the ſquare root of .079577, it will be .2821, which is the ſide of the ſquare equal to the circle whoſe circumference is 1.

If the ſide of a ſquare within a circle be required, if you ſquare the ſemidiameter and double that ſquare, and out of that ſum extract the ſquare root, that ſhall be the ſide of the ſquare which may be inſcribed in that circle: ſo, if the diameter of the circle be 1, then the half is .5; which ſquared is .25, and this doubled is .5, whoſe ſquare root is .7071, the ſide of the ſquare inſcribed.

Again, if the diameter of a globe be 1, to find the ſolidity. In Sect. XI. Chap. II. it is demonſtrated, that the globe is $\frac{2}{3}$ of a cylinder of the ſame diameter and altitude: thus, if the cylinder's diameter be 1, and its altitude or length be alſo 1, find the ſolidity thereof, and take $\frac{2}{3}$ of it, and that will be the ſolidity of the globe required. Now, if the diameter be 1, the area of the circle or baſe of the cylinder, is .7854 (as is above ſhewn) which multiplied by 1, the altitude of the cylinder, and the product is alſo .7854, the ſolidity of the cylinder, $\frac{2}{3}$ whereof is .5236, which is the ſolidity of the globe whoſe diameter is 1.

From what has been ſaid, the reader may eaſily perceive how all other proportional numbers are found, and may examine them at his pleaſure.

I ſhall not enlarge any farther upon the matter, but leave the book to ſpeak for itſelf; and if it prove beneficial to the ingenious practitioners, I have my deſire. So, wiſhing my ingenious reader good ſucceſs in his endeavours, not doubting but he will reap profit hereby; which that he may, is the hearty deſire of his well-wiſher,

W. HAWNEY.

THE

Complete Meaſurer.

PART I.

CHAP. I.

Notation of DECIMALS.

A Decimal fraction is an artificial way of ſetting down and expreſſing natural or vulgar fractions, as whole numbers: And whereas the denominators of vulgar fractions are diverſe, the denominators of decimal fractions are always certain; for a decimal fraction hath always for its denominator an unit, with a cipher or ciphers annexed to it, and muſt therefore be either 10, 100, 1000, 10000, *&c.* And therefore in writing down a decimal fraction there is no neceſſity of writing down the denominator; for by bare inſpection it is certainly known; it conſiſting of an unit, with as many ciphers annexed to it as there are places (or figures) in the numerator.

Example. This decimal fraction $\frac{25}{100}$ may be written thus, .25, its denominator being known to be an unit with two ciphers; becauſe there are two figures in the numerator. In like manner, $\frac{125}{1000}$ may be thus written, .125, $\frac{75}{1000}$ thus, .075; and $\frac{3575}{10000}$ thus, .3575, and $\frac{65}{10000}$ thus, .0065

As whole numbers increaſe in a decuple, or tenfold proportion, towards the left hand, ſo, on the contrary, decimals decreaſe towards the right hand, in a decuple proportion, as in the following ſcheme.

Tens of Millions	Millions	Hundreds of Thouſands	Tens of Thouſands	Thouſands	Hundreds	Tens	Units	Tenth Parts	Hundredth Parts	Thouſandth Parts	Ten Thouſandth Parts	Hundred Thouſandth Parts	Millionth Parts, *&c.*
7	6	5	4	3	2	1	0.	1	2	3	4	5	6

Hence it appears, that ciphers put on the right hand of whole numbers, do encreaſe the value of thoſe numbers in a decuple (or tenfold) proportion: but being annexed to the right hand of a decimal fraction, do neither increaſe nor decreaſe the value, thereof: So $\frac{2500}{10000}$ is equivalent $\frac{25}{100}$ or .25. And on the contrary, tho' in whole numbers, ciphers prefixed before them, do neither increaſe nor diminiſh the value; yet ciphers before a decimal fraction, do diminiſh its value in a decuple proportion: For .25 if you prefix a cipher before it, becomes $\frac{025}{1000}$, or .025; and .125 is $\frac{00125}{100000}$ by prefixing two ciphers before it, thus, .0025. And therefore, when you are to write a decimal fraction, whoſe denominator hath more ciphers than there are figures in the numerator, they muſt be ſupplied by prefixing ſo many ciphers before the figures of your numerator; as ſuppoſe $\frac{19}{1000}$ were to be written down without its denominator; here, becauſe there are three ciphers in the denominator, and but two figures in the numerator, therefore prefix a cipher before 19, and ſet it down thus, .019.

The integers are ſeparated from the decimals ſeveral ways according to mens fancies; but the beſt and moſt uſual way is by a point or period; and if there be no whole number, then a point before the fraction is ſufficient. Thus, if you were to write down 317 $\frac{217}{1000}$ it may be thus expreſſed, 317.217; and 59 $\frac{25}{10000}$ thus, 59.0025; and $\frac{75}{100000}$ thus, .0075, &c.

CHAP. II.

Reduction of Decimals.

IN *Reduction of Decimals* there are three caſes; 1ſt, To reduce a vulgar fraction to a decimal. 2dly, To find the value of a decimal in the known parts of coin, weights, meaſures, &c. 3dly, To reduce coin, weights, meaſures, &c. to a decimal. Of theſe in their order.

I. *To reduce a Vulgar Fraction to a Decimal.*

The RULE.

As the denominator of the given fraction is to its numerator, ſo is an unit (with a competent number of ciphers annexed) to the decimal required.

Therefore, if to the numerator given, you annex a competent number of ciphers, and divide the reſult by the denominator, the quotient is the decimal equivalent to the vulgar fraction given.

Example. 1. Let $\frac{1}{4}$ be given, to be reduced to a decimal of two places, or having 100 for its denominator.

To 3 (the numerator given) annex two ciphers, and it makes 300; which divide by the denominator, 4 and the quotient is .75, the decimal required, and is equivalent to $\frac{3}{4}$ given.

NOTE, That ſo many ciphers as you annex to the given numerator, ſo many places muſt be pricked off in the decimal found; and if it ſhall happen that there are not ſo many places of figures in the quotient, the deficiency muſt be ſupplied, by prefixing ſo many ciphers before the quotient figures, as in the next *Example*.

Example 2. Let $\frac{3}{573}$ be reduced to a decimal having ſix places.

To the numerator annex ſix ciphers, and divide by the denominator, and the quotient is .5235; but it was required to have ſix places, therefore you muſt prefix two ciphers before it, and then it will be .005235, which is the decimal required, and is equivalent to $\frac{3}{573}$.

See the work of theſe two *Examples*.

```
4) 3.00(.75          573) 3.000000(5235
   28                            ....
  ------                  ------------
   20                     1350
   20                       2040
  ------                      3210
   ..                     ------------
                                345
```

In the ſecond Example there remain 345, which remainder is very inſignificant, it being leſs than $\frac{1}{1000000}$ part of an unit, and therefore is rejected.

II. *To*

II. *To find the value of a Decimal in the known parts of Money, Weight, Meaſures,* &c.

The RULE.

Multiply the given decimal by the number of parts in the next inferior denomination, and from the product prick off ſo many places to the right hand, as there were places in the decimal given; and multiply thoſe figures pricked off by the number of parts in the next inferior denomination, and prick off ſo many places as before, and ſo continue to do, till you have brought it to the loweſt denomination required.

Example 1. Let .7565 of a pound ſterling be given to be reduced to ſhillings, pence and farthings.

Multiply by 20, by 12 and 4, as the rule directs, and always prick off four places to the right hand, and you will find it will make 15*s.* 1*d.* 2*q.*

See the work.

```
      .7565
         20
  s.  -----
  15. 1300
         12
  d.  -----
   1. 5600
          4
  q.  -----
   2. 2400
```

A more compendious way of finding the Value of the decimal of a Pound Sterling.

Double the firſt figure, (or place of primes) and it makes ſo many ſhillings; and if the next figure (or place of ſeconds) be 5, or more than 5, for the 5 add another ſhilling to the former ſhillings; then for every unit in the ſecond place count ten, and to that add the figure in the third place, and reckon them ſo many farthings, but if they make above 13, abate 1, and if it be above 38, abate 2, and add the remaining farthings to the ſhillings before found.

Example 1. Let .695 of a pound be reduced to ſhillings, pence, and farthings.

Firſt double your 6, and it makes 12*s.* then take 5 out of 9 and for that reckon another ſhilling, and it makes 13*s.* and the four remaining is 4 tens, and the 5 makes 45, which being above 38, you muſt therefore caſt away 2, and the reſt 43 farthings, which is 10*d.* ¾. So the anſwer is 13*s.* 10*d.* ¾.

	l.	*s.*	*d.*
So the value of	725=	14	6
And the value of	878=	17	6¾
And the value of	417=	8	4

And ſo of any other.

Let .59755 of a pound troy be reduced to ounces, penny-weights and grains.

Multiply by 12, by 20 and by 24, and always prick off five places towards the right hand, and you will find the anſwer to be 7*oz* 3 *pwt.* 10 *gr. ferè.*

See the Work.

```
   .59755
       12
  -------
  7.17060
       20     oz. pwt. gr.
  ------- Facit 7   3  9.888
  3.41200
       24
  -------
   164800
   82400
  -------
  9 88800
```

Let .43569 of a tun, be reduced to hundreds, quarters and pounds.

Multiply by 20, by 4, and by 28, and the anſwer will be 8*C.* 2*qrs.* 14*lb. ferè.*

.43569
20
———
8.71380
4
———
2.85520
28
———
23.94560

	C.	*qrs.*	*lb.*
Facit	8	2	23 .9456

Let .9595 of a foot be reduced into inches and quarters.

.9595
12
———
11.5140 *Facit* 11 inches, 2 quarters
4
———
2.0560

III. *To reduce the known parts of money, weight, meaſure, &c. to a Decimal.*

The R U L E.

To the number of parts of the leſſer denomination given, annex a competent number of ciphers, and divide by the number of ſuch parts that are contained in the greater denomination, to which the decimal is to be brought; and the quotient is the decimal ſought.

Example 1. Let 6*d.* be reduced to the decimal of a pound.

To 6 annex a competent number of ciphers (ſuppoſe 3) and divide the reſult by 240 (the pence in a pound) and the quotient is the decimal required.

240).6000(.25

1200

Facit 0.25.

. . . .

Example 2. Let 3*d*. ¼ be reduced to the decimal of a pound having ſix places.

In 3*d*. ¼ there are 15 farthings; therefore to 15 annex ſix ciphers (becauſe there are to be ſix places in the decimal required) and divide by 960 (the farthings in a pound) and the quotient is .015625.

96|0)15.00000|0(.015625.

. . . . .

540
600
240
480

. . .

Example 3. Let 3¼ inches be reduced to the decimal of a foot, conſiſting of four places.

In 3¼ inches there are 13 quarters: therefore to 13 annex four ciphers, and divide by 48, (the quarters in a foot) and the quotient is .2708.

48)13.0000(.2708

340
400

16

Example 4. Let 9*C.* 1*qr.* 16*lb.* be reduced to the decimal of a tun, having ſix places.

C. *qr.* *lb.*
9 1 16 224|0)1052.00000|0(469642
4 . . . . .
37 *qrs.* 15600
28 21600
14400
302 *Facit* .469642. 9600
75 6400
1052 pounds. 1920

CHAP. III.

Addition of DECIMALS.

ADDITION of Decimals is performed the ſame way as addition of whole numbers, only you muſt obſerve to place your numbers right, that is, units under units, primes under primes, ſeconds under ſeconds, &c.

Example. Let 317.25, 17.125, 275.5, 47.3579, and 12.75 be added together into one ſum.

317.25
17.125
275.5
47.3579
12.75

Sum 669.9829

This is ſo plain, that more Examples I think needleſs.

CHAP.

CHAP. IV.

Subtraction of DECIMALS.

SUBTRACTION of Decimals is performed likewiſe the ſame way as in whole numbers, reſpect being had to the right placing the numbers (as in Addition) as in the following examples.

(1)		(2)	
From	212.0137	From	201.1250
Subtr.	31.1275	Subtr.	5.5785
Reſts	180.8862	Reſts	195.5465
Proof	212.0137	Proof	201.1250

(3)		(4)	
From	2051.315	From	30.5
Subtr.	79.172	Subtr.	7.2597
Reſts	1972.143	Reſts	23.2403
Proof	2051.315	Proof	30.5

NOTE, If the number of places in the decimals be more in that which is to be ſubtracted, than in that which you ſubtract from, you muſt ſuppoſe ciphers to make up the number of places; as in the fourth Example.

CHAP. V.

Multiplication of DECIMALS.

MULTIPLICATION of Decimals is alſo performed the ſame way as Multiplication of whole numbers; but to know the value of the product, obſerve this

RULE.

" Cut off or ſeparate by a comma, or point, ſo
" many decimal places in the product, as there are
" places of decimals in both factors, *viz.* in the
" multiplicand and multiplier which I ſhall farther
" explain in the following

Examples.

Let 3.125 be multiplied by 2.75; multiply the numbers together, as if they were whole numbers, and the product is 8.59375: And becauſe there were three places of decimals pricked off in the multiplicand, and two places in the multiplier, therefore you muſt prick off five places of decimals in the product, as you may ſee by the work.

```
  3.125
   2.75
 ------
  15625
 21875
 6250
 ------
 8.59375
```

Let 79.25 be multiplied by .459.

In this Example, becauſe two places of decimals are pricked off in the multiplicand, and three in the multiplier, therefore there muſt be five pricked off in the product.

$$\begin{array}{r}
79.25\\
.459\\
\hline
71325\\
39625\\
31700\\
\hline
36.37575
\end{array}$$

Let .135272 be multiplied by .00425.

In this Example, becauſe in the multiplicand are ſix decimal places, and in the multiplier five places, therefore in the product there muſt be eleven places of decimals; but when the multiplication is finiſhed, the product is but 57490000, *viz.* only eight places; therefore, in this caſe, you muſt prefix three ciphers before the product figures to make up the number of eleven places; ſo the true product will be .00057490600.

$$\begin{array}{r}
.135272\\
.00425\\
\hline
676360\\
270544\\
541088\\
\hline
.00057490600
\end{array}$$

More Examples for practice.

.001472
.1045

7360
5888
1472

.0001538240

279.25
.445

139625
111700
111700

124.26625

4.443
15.98

35544
39987
22215
4443

70.99914

7.3564
.0126

441384
147128
73564

.09269064

.017532
347

122724
70128
52596

6.083604

32.0752
0.325

1603760
641504
962256

1.04244400

20.0292
35.45

1001455
801164
1001455
600873

710.031595

.75432
.0356

452592
377160
226296

.026853792

Contracted Multiplication of Decimals.

Becauſe in Multiplication of decimal parts and mixed numbers, there is no need to expreſs all the figures of the product, but in moſt caſes, two, three or four places of decimals will be ſufficient; therefore, to contract the work, obſerve this following

RULE.

Write the Units place of the multiplier under that place of the multiplicand, whoſe place you intend to keep in the product; then invert the order of all the other figures, that is, write them all the contrary way; then, in multiplying, always begin at that figure in the multiplicand which ſtands over the figure you are then multiplying withal, and ſet down the firſt figure of each particular product directly one under the other; but yet a due regard muſt be had to the increaſe ariſing from the figures on the right hand of that figure in the multiplicand which you begin to multiply at. This will appear more plain by Examples.

Example 1. Let 2.38645 be multiplied by 8.2175, and let there be only four places retained in the decimals of the product.

Firſt, according to the directions, write down the multiplicand, and under it write the multiplier, thus; place the 8 (being the units place of the multiplier) under 4, the fourth place of decimals in the multiplicand, and write the reſt of the figures quite contrary to the uſual way, as in the following work. Then begin to multiply, firſt the five which is left out, (only with regard to the increaſe which muſt be carried from it) ſaying, 8 times 5 is 40, carry 4 in your mind, and ſay 8 times 4 is 32, and 4 I carry, is 36; ſet down 6 and carry 3; and proceed thro' the reſt of the figures as in common multiplication: Then begin to multiply with 2, ſaying, 2 times 4 is 8, for which I carry 1, (becauſe it is above 5) and ſay 2 times 6 is 12, and one that I carry is 13, ſet down 3 and carry 1, and proceed thro' the reſt of the figures: Then multiply with 1, and ſay once 6 is 6, for which carry one, and ſay once 8 is 8, and 1 is 9; ſet down 9, and proceed: Then multiply with 7, ſaying 7 times 8 is 56, for which carry 6, (becauſe it is above 55) and ſay 7 times 3 is 21, and 6 that I carry is 27; ſet down 7 and carry 2, and proceed: Then multiply with 5, ſaying 5 times 3 is 15, for which carry 2, and ſay 5 times 2 is 10, and 2 I carry is 12, which ſet down and add all the products together, and the total product will be 19.6107, See the work.

2.38645
5712.8

190916
4773
239
167
12

19.6107

NOTE, That in multiplying the figure left out every time next the right hand in the multiplicand, if the product be 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2, and if 25, or upwards to 30, carry 3, *&c.*

I have here ſet down the work of the laſt Example, wrought by the common way, by which you may ſee both the reaſon and excellency of this way, all the figures on the right hand of the line being wholly omitted.

2.38645
8.2175

11|93225
167|0515
238|645
4772|90
190916|0

19.6106|52875

Example 2. Let 375.13758 be multiplied by 16.7324, ſo that the product may have but four places of decimals.

Firſt ſet 6, the unit's place of the multiplier, under 5, being the fourth place of decimals in the multiplicand, (becauſe four places of Decimals were to be pricked off) and write all the reſt of the figures backward; then multiply all the figures of the multiplicand by 1, after the common way; then begin with the ſecond figure of the multiplier, 6, ſaying 6 times 8 is 48, for which I carry 5; (in reſpect of the 8 left out) and 6 times 5 is 30, and 5 that I carry is 35; ſet down 5 and carry 3, and proceed after the common method: Then begin with 7, the third figure of the multiplier, and ſay 7 times 5 is 35, for which carry 4, and ſay 7 times 7 is 49 and 4 I carry is 53; ſet down three under the firſt, and carry 5, and proceed as before: Then begin with 3, the fourth figure of the multiplier, and ſay 3 times 7 is 21, carry 2, and ſay 3 times 3 is 9, and 2 I carry is 11, ſet down 1 and carry 1, and proceed as before: Then begin with 2, the fifth figure, and ſay 2 times 3 is 6, for which I carry 1, and ſay 2 times 1 is 2, and 1 I carry is 3, ſet down 3, and ſay 2 times 5 is 10, ſet down 0 and carry 1, and proceed as before: Then begin with 4, the laſt figure of the multiplier, and ſay 4 times 1 is 4, for which I carry nothing, becauſe it is leſs than 5; then ſay 4 times 5 is 20; ſet down 0 and carry 2, and proceed thro' the reſt of the figures of the multiplicand: Then add all up together, and the product is 6276 9520.

See the Work.

375.13758	the multiplicand.
4237.61	the multiplier reverſed.
3751.3758	the product with 1.
22508255	the product with 6 increaſed with 6 × 8.
2625963	the product with 7 increaſed with 7 × 5.
112541	the product with 3 increaſed with 3 × 7.
7503	the product with 2 increaſed with 2 × 3.
1500	the product with 4 increaſed with 0.
6276.9520	the product required.

Let the ſame Example be repeated, and let only one place in decimals be pricked off.

375.13758 the multiplicand.
4237.61 the multiplier inverted.

37514 the product by 1 with the increaſe of 1 × 7
22508 the product with 6 increaſed with 6 × 3.
2626 the product with 7 increaſed with 7 × 1.
113 the product with 3 increaſed with 3 × 5.
7 the product with 2 increaſed with 2 × 7.
1 the increaſe only of 4 × 3.

6276.9 the product is the ſame as before.

More Examples for practice.

Multiply 395.3756 by .75642, and prick off four places in decimals.

395.3756 the multiplicand.
24657. the multiplier reverſed.

2767629 the product by 7 increaſed with 7 × 6.
197688 the product by 5 increaſed with 5 × 5.
23722 the product by 6 increaſed with 6 × 7.
1581 the product by 4 increaſed with 4 × 3.
79 the product by 2 increaſed with 2 × 5.

299.0699 the product required.

Let the ſame Example be repeated, and let there be only one place of decimals.

395·3756
24657.
2767 the product by 7 increaſed with 7 × 3.
198 the product by 5 increaſed with 5 × 5.
24 the product by 6 increaſed with 6 × 9 + 6 × 3
2 the increaſe of 4 × 9 + 4 × 3.

299.1 the product.

Characters and their Signification.

Note, That this mark + ſignifies Addition: 8 + 5, that is 8 more 5, or 8 added to 5; and 8 + 3 + 7, denotes theſe numbers are to be added into one ſum.

This mark — ſignifies ſubtraction; as 9 — 4 ſignifies that 4 is to be taken from 9.

This mark × ſignifies multiplication; as 7 × 5, ſignifies that 7 is to be multiplied into 5.

This ÷ ſignifies diviſion; as 12 ÷ 4 ſignifies that 12 is to be divided by 4.

This mark = ſignifies equality, or equation, that is, when = is placed between numbers or quantities it denotes them to be equal; as 7 + 5 = 82, that is, 7 more 5 is equal to 12; and 15 — 7 = 8, that is, 15 leſs by 7, is equal to 8, or ſubtract 7 from 15 and there remains 8.

This mark : : is the ſign of proportion, or the Golden Rule, it being always placed between the two middle terms or numbers in proportion, thus, 4 : 20 : : 6 : 30; to be thus read, as 4 is to 20, ſo is 6 to 30.

CHAP.

CHAP. VI.

Diviſion of DECIMALS.

DIVISION of decimals is performed at the ſame manner as diviſion of whole numbers; but to know the value of the denomination of the quotient is the only difficulty; for the reſolving of which obſerve either of the following

RULES.

I. The firſt figure in the quotient muſt be of the ſame denomination with that figure in the dividend which ſtands (or is to be ſuppoſed to ſtand) over the unit's place in the diviſor, at the firſt ſeeking.

II. When the work of diviſion is ended, count how many places of decimal parts there are in the dividend more than in the diviſor, for that exceſs is the number of places which muſt be ſeparated in the quotient for decimals: But if there be not ſo many figures in the quotient, as is the ſaid exceſs, that deficiency muſt be ſupplied with ciphers in the quotient, prefixed before the ſignificant figures thereof, towards the left hand, with a point before them; ſo ſhall you plainly diſcover the value of the quotient.

Theſe following directions ought alſo to be carefully obſerved.

If the diviſor conſiſts of more places than the dividend, there muſt be a competent number of ciphers annexed to the dividend, to make it conſiſt of as many (at leaſt) or more places of decimals than the diviſor; for the ciphers added muſt be reckoned as decimals.

Conſider whether there be as many decimal parts in the dividend as there are in the diviſor; if there be not, make them ſo many or more, by annexing of ciphers.

In dividing of whole or mixed numbers, if there be a remainder, you may bring down more ciphers, and by continuing your diviſion, carry the quotient to as many places of decimals as you pleaſe.

Theſe things being conſidered, I ſhall proceed to the practice of diviſion of decimals, which I ſhall endeavour to explain in as familiar and eaſy a method as poſſible.

Example 1. Let 48 be divided by 144.

In this example the diviſor 144 is greater than the dividend 48; therefore, according to the directions, above, I annex a competent number of ciphers, (*viz.* four) with a point between them, and divide according to the uſual way,

```
144) 48.0000 (.3333
     ————————
      480
       480
        480
        ———
         48
```

But firſt in ſeeking how often 144. in 480, (the firſt three figures of the dividend) I find the unit's place of the diviſor to fall under the firſt place of decimals; therefore the firſt figure in the quotient is in the firſt place of decimals: Or by the ſecond rule, there being four places of decimals in the dividend, and none in the diviſor, ſo the exceſs of decimal places in the dividend, above that in the diviſor, is four ſo that when the diviſion is ended, there muſt be four places of decimals in the quotient.

See the Work.

Example 2. Let 217.75 be divided by 65.

Firſt, in ſeeking how often 65 in 217, (the firſt three figures of the dividend) I find the unit's place of the diviſor to fall under the unit's place of the dividend; therefore the firſt figure in the quotient will be units, and all the reſt decimals. Or, by the ſecond rule, there being two places of decimals in the dividend, and no decimals in the diviſor, therefore the exceſs of decimal places in the dividend, above the diviſor is two; ſo when the diviſion is ended, ſeparate two places in the quotient towards the right hand by a point.

See the Work.

65) 217.75 (3.35
. . .

227
325

. . .

Example 3. Let 267.15975 be divided by 13.25.

13.25)267.15975(20.163
. . . . .

2159
8347
3975

. . . .

In this Example 3, the unit's place of the diviſor, falls under 6, the ten's place of the dividend; therefore (by the firſt rule) the firſt figure in the quotient is tens: Or by the ſecond rule, the exceſs of decimal places in the dividend, above the diviſor, is three; there being five places of decimals in the dividend, and but two in the diviſor, ſo there muſt be three places of decimals in the quotient.

Example 4. Let 15.675159 be divided by 375.89.

375.89) 15.675159 (0417
. .

63955
263669

546

In this Example 5, the unit's place of the diviſor falls under 7, the ſecond place of decimals in the dividend; therefore (by the firſt rule) the firſt figure in the quotient is in the ſecond place of decimals, ſo that you muſt put a cipher before the firſt figure in the quotient, and by the ſecond rule, the exceſs of decimal places in the dividend, above the number of decimal places in the diviſor, is 4; for the decimal places in the dividend is 6, and the number of places in the diviſor but two; therefore there muſt be four places of decimals in the quotient. But the diviſion being finiſhed after the common way, the figures in the quotient are but three, therefore you muſt prefix a cipher before the ſignificant figures.

Example 5. Let 72.1564 be divided by .1347.

```
.1347)72.1564(535.68
          ...
       --------
       4806
        7654
          9190
          11080
       --------
            304
```

In this Example, the diviſor being a decimal, the firſt figure thereof falls under the ten's place in the dividend; therefore the units (if there had been any) ſhould fall under the hundred's place in the dividend, and ſo the firſt figure in the quotient is hundreds: And by the ſecond rule, there being four places of decimals in the dividend, and as many in the diviſor, ſo the exceſs is nothing; but in dividing I put two ciphers to the remainders, and continue the diviſion to two places farther; ſo I have two places of decimals.

See the Work.

Example 6. Let .125 be divided by .0457.

```
.0457).1250000(2.735
         . . . .
         914
        -----
        3360
        3199
        -----
         1610
         1371
         -----
          2390
          2285
          -----
           105
```

In this Example, the unit's place of the diviſor (if there had been any) would fall under the unit's place of the dividend; therefore the firſt figure of the quotient is units: And by the ſecond rule, there being ſeven places of decimals in the dividend, and but four places in the diviſor, ſo the exceſs is three; therefore there muſt be three places of decimals in the quotient.

I ſhall ſet down only the work of ſome few Examples more, and ſo proceed to *Contracted Diviſion.*

```
.00456).0000059791 (.00131
              . .
             -----
             1419
              511
             -----
               55
```

Let 1 be divided by 282, *&c.*

```
282) 1.0000000 (.0035461 fere.
         .....
      ----------
      1540
        1300
          1720
            280

   325) .400000 (1.2307
              ...
        ----------
             750
             1000
               2500
        ----------
                225

.042) 495.00000 (11785.71
          ......
       ----------
         75
         330
           360
            240
             300
               60
       ----------
               18
```

Diviſion of DECIMALS *contracted.*

IN Diviſion of decimals the common way, when the diviſor hath many figures, and it is required to continue the diviſion till the value of the remainder be but ſmall, the operation will ſometimes be large and tedious, but may be excellently contracted by the following method.

The RULE.

By the firſt rule of this Chapter, *(pag.* 19.*)* find what is the value of the firſt figure in the quotient; then, by knowing the firſt figure's denomination, you may have as many or as few places of decimals as you pleaſe, by taking as many of the left hand figures of the diviſor as you think convenient for the firſt diviſor; and then take as many figures of the dividend as will anſwer them; and in dividing, omit one figure of the diviſor at each following operation.

A few *Examples* will make it plain.

Example 1. Let 721.17562 be divided by 2.25743 and let there be three places of decimals in the quotient.

```
2.25743) 721.175|62 (319.467
         677229
         ------
          43946
          22574
          -----
          21372
          20317
          -----
           1055
            903
           ----
            152
            135
            ---
             17
             15
             --
              2
```

In this Example, the unit's place of the diviſor falls under the hundred's place in the dividend; and it is required that three places of decimals be in the quotient; ſo there muſt be ſix places in all, that is, three places of whole numbers, and three places of decimals: Then, becauſe I can have the diviſor in the firſt ſix figures of the dividend, I cut off the 62 with a daſh of the pen, as uſeleſs; then I ſeek how oft the diviſor in the dividend, and the anſwer is three times; put three in the quotient, and multiply and ſubtract as in common diviſion, and the remainder is 43946; then prick off the 3 in the diviſor, and ſeek how oft the remaining figures may be had in 43946, the remainder, which can be but once; put 1 in the quotient, and multiply and ſubtract, and the next remainder is 21372: Then prick off the 4 in the diviſor, and ſeek how often the remaining figures may be had in 2372, which will be nine times; put 9 in the quotient; multiply thus, ſaying 9 times 4 is 36; for which I carry 4, (in reſpect of the 4 laſt pricked off) and 9×7 is 63, and 4 is 67; ſet down 7 and carry 6, and ſo proceed till the diviſion be finiſhed, always reſpecting the increaſe made from the figures pricked off. Obſerve the work, which will better inform you than many words.

2.25743) 721.17562 (319.467

· · ·

$$\begin{array}{r|l}
677229 & \\ \hline
43946 & 6\\
22574 & 3\\ \hline
21372 & 32\\
20316 & 87\\ \hline
1055 & 450\\
902 & 972\\ \hline
152 & 4780\\
135 & 4458\\ \hline
17 & 03220\\
15 & 80201\\ \hline
1 & 23019
\end{array}$$

I have ſet down the work of this laſt Example at large, according to the common way, that thereby the learner may ſee the reaſon of the rule, all the figures on the right ſide of the perpendicular line being wholly omitted.

Example 2. Let 5171.59165 be divided by 8.758615, and let it be required that four places of decimals be pricked off in the quotient.

```
8.758615)5171.5916|5(590.4577
 ......
           43793075
           --------
            7922841
            7882754
            -------
              40087
              35034
              -----
               5053
               4379
               ----
                674
                613
                ---
                 61
                 61
                 --
                 ..
```

In this Example, I can't have 8, the firſt figure in the diviſor, in 5, the firſt figure of the dividend, ſo that the unit's place of the diviſor falls under the hundred's place of the dividend, ſo that there will be ſeven figures in the quotient, that is, three of whole numbers, and four of decimals; therefore there muſt be 7 figures in the diviſor, becauſe the number of places in the diviſor and quotient will be equal, and there muſt be eight places in the dividend, ſo that I cut off the figure 5 with a daſh, as uſeleſs. Thus having proportioned the dividend to the diviſor, and both to the number of places, or figures deſired in the quotient, I proceed to divide as before, ſaying, how often 8 in 51, which will be 5 times, put 5 in the quotient, and multiply and ſubtract, and the remainder is 7922841; then I prick off the firſt figure in the diviſor, 5, and ſeek how often the remaining figures of the diviſor in the aforeſaid remainder, which I find 9 times; put 9 in the quotient, and multiply thereby, ſaying, 9 times 5 (the figures pricked off) are 45, for which I carry 5, and ſay 9 times 1 is 9; and 5 I carry is 14; ſet down 4 and carry 1, and proceed to multiply the reſt of the figures, and ſubtract, and the remainder will be 40087. Then prick off the figure 1, and ſeek how often 87586 in the remainder 40087, the anſwer will be 0; ſo put 0 in the quotient, and prick off the figure 6, and ſeek how often 8758 in 40087, which will be 4 times; put 4 in the quotient, and multiply, ſaying, 4 times 6 (the figure laſt pricked off) is 24, for which I carry 2, and ſay 4 times 8 is 32, and 2 I carry is 34; ſet down 4 and carry 3; multiply the reſt of the figures, and ſubtract as before; and ſo proceed after the ſame manner, until all the figures of the diviſor be pricked off to the laſt figure.

See the Work.

Example. 3. Let 25.1367 be divided by 217.3543, and let there be five places of decimals in the quotient.

In this Example, 7, the unit's place of the diviſor falls under 1, the firſt place of the decimals, therefore the firſt figure of the quotient is in the firſt place of decimals, ſo the quotient will be all decimals: Then, becauſe the quotient figures, and the figures of the diviſor will be of an equal number, daſh off the 43 in the diviſor, and the 7 in the dividend, as uſeleſs, and divide as before.

$$\begin{array}{r}
217.35|43)25.136|7(.11564 \\
21735 \\
\hline
3401 \\
2174 \\
\hline
1227 \\
1087 \\
\hline
140 \\
130 \\
\hline
10 \\
8 \\
\hline
2
\end{array}$$

Altho' I have hitherto given directions for proportioning the diviſor and dividend, ſo as to bring into the quotient what number of decimals you pleaſe, yet there is no abſolute neceſſity for it; but you may carry on your diviſion to what degree you pleaſe, before you begin to prick off the figures of the diviſor, in order to contract the work, as in the following Examples, where it is not required to prick off any determinate number of decimals, but it may be done according to diſcretion.

```
2.756756) 7414.76717 (2689.67118
  ......          ...
          5513512
          --------
          19012551
          16540536
          --------
          24720157
          22054048
          --------
           2666109
           2481080
           -------
            185029
            165405
            ------
             19624
             19297
             -----
               327
               276
               ---
                51
                28
                --
                23
                22
                --
                 1
```

```
12.34254)514.75498(41.705757
     ...  4937016
          -------
          2105338
          1234254
          -------
           871084
           863978
           ------
             7106
             6171
             ----
              935
              864
              ---
               71
               62
               --
                9
                8
                -
                1
```

CHAP. VII.

Extraction of the SQUARE-ROOT.

If a Square *Number be given.*

TO find the root thereof; that is, to find out ſuch a number, as being multiplied into itſelf, the product ſhall be equal to the number given, ſuch operation is called, *The Extraction of the* SQUARE ROOT; which to do, obſerve the following directions.

1ſt, You muſt point your given number, that is, make a point or prick over the unit's place, another upon the hundred's, and ſo upon every ſecond figure throughout.

2dly, Then ſeek the greateſt ſquare number in the firſt point towards the left hand, placing the ſquare number under the firſt point, and the root thereof in the quotient, and ſubtract the ſaid ſquare number from the firſt point, and to the remainder bring down the next point, and call that the reſolvend.

3dly, Then double the quotient, and place it for a diviſor, on the left hand of the reſolvend, and ſeek how often the diviſor is contained in the reſolvend, (reſerving always the unit's place,) and put the anſwer in the quotient, and alſo on the right hand ſide of the diviſor; then multiply by the figure laſt put in the quotient, and ſubtract the product from the reſolvend (as in common diviſion) and bring down the next point to the remainder (if there be any more) and proceed as before.

A Table *of Squares and Cubes, and their Roots.*

Root	1	2	3	4	5	6	7	8	9
Squa.	1	4	9	16	25	36	49	64	81
Cube	1	8	27	64	125	216	343	512	729

Example 1. Let 4489 be a number given, and let the ſquare root thereof be required.

$$\begin{array}{rrl} & \overset{.}{4}4\overset{.}{8}9 & 67 \\ & 36 & \\ \hline 127 & 889 & \text{Reſolvend} \\ & 889 & \text{Product.} \\ \hline & \ldots & \end{array}$$

Firſt, point the given number, as before directed, then (by the little Table aforegoing) ſeek the greateſt ſquare number in 44, (the firſt point to the left hand) which you will find to be 36, and 6 the root, put 36 under 44, add 6 in the quotient, and ſubtract 36 from 44, and there remains 8: Then to that 8 bring down the other point 89, placing it on the right hand, ſo it makes 889 for a reſolvend; then double the quotient 6, and it makes 12, which place on the left hand for a diviſor, and ſeek how often 12 in 88 (reſerving the unit's place) the anſwer is 7 times, which put in the quotient; and alſo on the right hand ſide of the diviſor, and multiply 127 by 7, as in common diviſion) and the product is 889, which ſubtracted from the reſolvend, there remains nothing, ſo is your work finiſhed, and the ſquare root of 4489 is 67, which root, if you multiply by itſelf, that is 67 by 67, the product will be 4489, equal to the given ſquare number, and proves the work to be right.

Example 2. Let 106929 be a number given, and let the ſquare root thereof be required.

• • •
106929(327
9

62) 169 Reſolvend.
124 Product.

647) 4529 Reſolvend.
4529 Product.

• • • •

Firſt, point your given number, as before directed, putting a point upon the units, hundreds and tens of thouſands; then ſeek what is the greateſt ſquare number in 10 (the firſt point) which by the little table you will find to be 9, and the root thereof; put 9 under 10, and 3 in the quotient; then ſubtract 9 out of 10, and there remains 1, to which bring down 69, the next point, and it makes 169 for the reſolvend;

then double the quotient 3, and it makes 6, which place on the left hand of the reſolvend for a diviſor, and ſeek how often 6 in 16, the anſwer is twice, put 2 in the quotient, and alſo on the right hand of the diviſor, making it 62: Then multiply 62 by the 2 you put in the quotient, and the product is 124; which ſubtract from the reſolvend, and there remains 45; to which bring down 29, the next point, and it makes 4529 for a new reſolvend; then double the quotient 32, and it makes 64, which place on the left ſide the reſolvend for a diviſor, and ſeek how oft 64 in 452, which you will find 7 times: put 7 in the quotient and alſo on the right hand of the diviſor, making it 647, which multiplied by 7 in the quotient, it makes 4529 which ſubtract from the reſolvend, there remains nothing: So 327 is the ſquare root of the given number.

Example 3. Let 2268741 be a ſquare number given, the root whereof is required.

```
                  .  .  .  .
                 2268741 (150623
                 1
           25)   126
                 125
           3006) 18741
                 18036
            30122) 70500
                   60244
          301243 1025600
                  .903729
         Remains .121871
```

Having pointed the given number as before directed, ſeek what is the greateſt ſquare number in the firſt point 2, which is 1; put 1, the ſquare under 2, and 1 the root thereof in the quotient; ſubtract 1 from 2, and there remains 1; to which bring down the next point 26, and ſet on the right hand, making it 126; double the 1 in the quotient, which makes 2 ſet 2 on the left hand for a diviſor, and aſk how often

ten 2 in 12, which will be five times; put 5 in the quotient and alſo on the right hand of the diviſor, making it 25; multiply (as in common diviſion) 25 by 5, and ſubtract the product, 125, from 126, and there remains 1; bring down the next point, 87, and it makes 187 for a new reſolvend, and double the 15 in the quotient, it makes 30 for a new diviſor; then ſeek how often 30 in 18, which you can't have; ſo that you muſt put 0 in the quotient, and alſo on the right hand of the diviſor, and bring down the next point, and it makes 18741 for another new reſolvend then ſeek how often 300 in 1874 which will be 6 times; put 6 in the quotient, and alſo on the right hand of the diviſor, multiply and ſubtract, and the remainder will be 705. Now, if you have a mind to find the value of the remainder, you may annex ciphers, by two at a time to the remainders, and ſo proſecute the work to what number of decimal parts you pleaſe; thus, to 705 annex two ciphers, and it will make 70500, and the quotient doubled, is 3012 for a diviſor; then ſeek how often 3012 in 7050 (rejecting the unit's place) which will be twice; put 2 in the quotient, and alſo on the right hand of the diviſor, and multiply and ſubtract as before, and the remainder will be 10256; to which annex two ciphers, and proceed as before, and you will get 3 in the quotient next. So the ſquare root of the given number is 1506.23, which being ſquared or multiplied by itſelf, and the laſt remainder added, will make the given number as follows:

1506.23
1506.23

451869
301246
903738
753115
150623

2268728 8129
The Remainder add ——— 12.1871

Proof 2268741.0000

Some

Some more Examples for Practice.

Example 1. $\dot{7}5\dot{9}6\dot{7}9\dot{6}$ (2756.228 Root.

4

47) 359

329

545) 3067

2725

5506) 34296

33036

55122) 126000

110244

551242) 1575600

1102484

5512448) 47311600

44099584

3212016

More Examples for Practice.

Example 2. 751417.5745 (866.84 Root.

64

166) 1114

966

1726) 11817

10356

17328) 146157

138624

173364) 753345

693456

59889

If the given number be a mixed number *viz.* conſiſting of a whole number and a decimal together, make the number of decimal places even, that is 2, 4, 6, 8, *&c.* that ſo there may a point fall upon the unit's place of the whole numbers, as in this laſt Example, and in that following.

Example 3. Let 656714.37512 be given to find the ſquare root.

656714.375120(810.379 Root,
64

161) 167
161

16203) 61437
48609

162067) 1282851
1134469

1620749 14838220
14586741

Remains 251479

In this Example there are five places of decimals: put a cipher to it, to make it even, that ſo there may a point fall upon 4, the unit's place.

To find the Square Root of a Fraction.

If it be a decimal fraction, the work differs nothing from the Examples foregoing, only you muſt be mindful to point your given number aright; for (as was before directed) the number of places muſt always be made even, and then begin to point at the right hand, as in whole numbers.

If it be a vulgar fraction, it muſt be reduced to a decimal, by the firſt rule of the ſecond Chapter.

I ſhall give an example or two in each caſe, and ſo conclude this chapter.

Let .125 be a decimal fraction given, whoſe ſquare root is required; and let it be required to have four places of decimals in the root.

```
         . . . .
        .12500000 (.3333
          9
        -----
    65) 350
        325
        -----
   703)2500
       2109
        ------
  7065) 39100
        35325
        ------
         3775
```

In this Example there muſt be five ciphers annexed, becauſe two places in the ſquare make but one in the root.

Let the Square Root of .00715 be required.

```
      .007150 (.084
        64
       ----
  164) 750
       656
       ----
        94
```

In this a cipher is added to make the places even.

Let $\frac{7}{8}$ be a vulgar fraction given, whoſe ſquare root is required.

```
                              . . . .
8) 7000                  ). 87500000 (.9354
   64                       81
   ----                     ----
    60                 183) 650
    56                      549
   ----                     ------
     40               1865) 10100
     40                      9325
    --                      -------
     ..              18704)  77500
                              74816
                            -------
                               2684
```

Reduce this $\frac{7}{8}$ to a decimal, it makes .875, to which annex ciphers, and extract the ſquare root, as if it was a whole number. So the root is .9354.

Let $\frac{3}{960}$ be a vulgar fraction, whoſe ſquare root is required.

```
                                    . . . . Root
96|0) 3.0000000|0              ).00312500 (.0559
          .....                    25
     288                          ----
     ---                      105)625
      120                         525
       96                        -----
      ---                    1109) 10000
       240                          9981
       192                         ------
      -----                            19
        480
        480
      -----
```

In extracting the root of this, becauſe the firſt point conſiſts of ciphers, there muſt be a cipher put firſt in the quotient.

To prove this rule, ſquare the root, and to the product add to the remainder, as was before directed. To ſquare a number is to multiply it by itſelf; and to cube it, is to multiply the ſquare of the number by the number itſelf.

CHAP. VIII.

Extraction of the Cube Root.

TO extract the Cube Root, is nothing elſe but to find ſuch a number, as being firſt multiplied into itſelf, and then into that product, produceth the given number; which to perform, obſerve theſe following directions.

1ſt, You muſt point your given number, beginning with the unit's place, and make a point or dot over every third figure towards the left hand.

2dly, Seek the greateſt cube number in the firſt point towards the left hand, putting the root thereof in the quotient, and the ſaid cube number under the firſt point, and ſubtract it therefrom, and to the remainder bring down the next point, and call that the reſolvend.

3dly, Triple the quotient, and place it under the reſolvend, the unit's place of this under the tens place of the reſolvend; and call this the triple quotient.

4thly, Square the quotient, and triple the ſquare, and place it under the triple quotient, the units of this under the ten's place of the triple quotient, and call this the triple ſquare.

5thly, Add theſe together, in the ſame order as they ſtand, and the ſum ſhall be the diviſor.

6thly, Seek how often the diviſor is contained in the reſolvend, rejecting the unit's place of the reſolvend, (as in the ſquare root) and put the anſwer in the quotient.

7thly, Cube the figure laſt put in the quotient, and put the unit's place thereof under the unit's place of the reſolvend.

8thly, Multiply the ſquare of the figure laſt put in the quotient, into the triple quotient, and place the product under the laſt, one place more to the left hand.

9thly, Multiply the triple ſquare by the figure laſt put in the quotient, and place it under the laſt, one place more to the left hand.

10thly, Add the three laſt numbers together, in the ſame order as they ſtand, and call that the ſubtrahend.

Laſtly, Subtract the ſubtrahend from the reſolvend, and if there be another point, bring it down in the remainder, and call that a new reſolvend, and proceed in all reſpects as before, from the beginning of the third ſtep to the end of this laſt.

Example 1. Let 314432 be a cubic number, whoſe root is required.

314432 216	)68 Root.
98432	Reſolvend.
18	Triple quotient of 6.
108	Triple ſquare of the quotient 6.
1098	Diviſor.
512	Cube of 8 the laſt figure of the Root.
1152	The ſquare of 8, by the triple quotient.
864	The triple ſquare of the quotient 6, by 8.
98432	The Subtrahend.

.....

After you have pointed the given number, ſeek what is the greateſt cube number in 314, the firſt point, which, by the former little table [page 34] you will find to be 216, which is the neareſt that is leſs than 314, and its root is 6; which put in the quotient, and 216 under 314, and ſubtract it therefrom, and there remains 98, to which bring down the next point, 432, and annex to 98, ſo will it make 98432 for the reſolvend; then triple the quotient 6, it makes 18, which write down, the unit's place, 8, under 3, the tens place of the reſolvend: Then ſquare the quotient 6, and triple that ſquare, and it makes 108, which write under the triple quotient, one place on the left hand; then add thoſe two numbers together, and they make 1098 for the diviſor; then ſeek how often the diviſor is contained in the reſolvend (rejecting the unit's place thereof) that is, how often 1098 in 9843, which is 8 times; put 8 in the quotient, and the cube thereof below the diviſor, the unit's place under the unit's place of the reſolvend : Then ſquare the 8 laſt put in the quotient, and multiply 64, the ſquare thereof, by the triple quotient 18, the product is 1152 ; ſet this under the cube of 8, the units of this under the tens of that.

Then multiply the triple ſquare of the quotient by 8, the figure laſt put in the quotient, the product is 864 ; ſet this down under the laſt quotient a place more to the left hand : then draw a line under thoſe three, and add them together, and the ſum is 98432, which is called the ſubtrahend, which being ſubtracted from the reſolvend, the remainder is nothing. Which ſhews the number to be a true cubic number, whoſe root is 68; that is if 68 be cubed it will make 314432.

For, if 68 be multiplied by 68, the product will be 4624, and this product multiplied again by 68, the laſt product is 314432 ; which ſhews the work to be right.

```
                68
                68
               ----
               544
              408
              -----
The Work.     4624
                68
             ------
             36992
            27744
            ------
The Proof   314432
```

Example. Let the cube root of 57353339 be required.

After you have pointed the given number, ſeek what is the greateſt cube number in 5, the firſt point, which (by the little table page 34) you will find to be 1, which place under 5, and 1, the root thereof, in the quotient, and ſubtract 1 from 5, and there remains 4, to which bring down the next point, it makes 4735 for the reſolvend: Then triple the 1, and it makes 3, and the ſquare of 1 is 1, and the triple thereof is 3; which ſet one under another, in their order, and added makes 33 for the diviſor: Seek how often the diviſor in the reſolvend, and proceed as in the laſt Example.

· · ·
5735339 (179 Root.
1

4735 Reſolvend.

3 Triple of the quotient 1. the firſt figure.
3 The triple ſquare of the quotient 1.

33 The diviſor.

343 The cube of 7, the ſecond figure of the root.
147 The ſquare of 7, mult. in the triple quot. 3.
21 The triple ſquare of the quotient, mult. by 7.

3913 The ſubtrahend.

822339 The new reſolvend.

51 The trip. of the quot. 17, the two firſt fig.
867 The triple ſquare of the quotient 17.

8721 Diviſor.

729 The cube of 9, the laſt figure of the root.
4131 The ſquare of 9, mult. by the trip. quot. 51.
7803 The triple ſquare of the quotient 867 by 9.

822339 The ſubtrahend.

· · · · · ·

In this Example, 33, the firſt diviſor, ſeems to be contained more than 7 times in 4735, the reſolvend;

but if you work with 9, or 8, you will find that the ſubtrahend will be greater than the reſolvend.

Some more Examples for Practice.

· · ·
32461759 (319 The root.
27

5461 Reſolvend.

9 The triple of 3.
27 The triple ſquare of 3.

279 The Diviſor.

1 The cube of 1, the ſecond figure.
9 The triple quotient by the ſquare of 1.
27 The triple ſquare mult. by 1, the ſecond fig.

2791 The ſubtrahend.

2670759 A new reſolvend.

93 The triple of 31.
2883 The triple ſquare of 31.

28923 The diviſor.

729 The cube of 9, the laſt figure.
7533 The ſquare of 9, by 93, the triple quot.
25947 The triple ſquare 2883 by 9.

2670759 The ſubtrahend.

·······

. . .
84604519(439 The Root.
64

20604 Refolvend.

12 The triple of 4.
48 The triple ſquare of 4.

492 The diviſor.

27 The cube of 3.
108 The ſquare of 3, by the triple quotient.
144 The triple ſquare by 3.

15507 The ſubtrahend.

5097519 Reſolvend.

129 The triple of 43.
5547 The triple ſquare of 43.

55599 The diviſor.

729 The cube of 9.
10449 The ſquare of 9, by 129.
49923 The triple ſquare by 9.

5097519 The ſubtrahend.

....

259697989(638
216

43697 Reſolvend.

18 The triple of 6.
108 The triple ſquare of 6.

1098 The diviſor.

27 The cube of 3, the ſecond figure.
162 The ſquare of 3 by 18.
324 The triple ſquare of 108 by 3.

34047 The ſubtrahend.

9650989 Reſolvend.

189 The triple of 63.
11907 The triple ſquare of 63.

119259 The diviſor.

512 The cube of 8.
12096 The ſquare of 8 by 189.
95256 The triple ſquare 11907 by 8.

9647072 The ſubtrahend.

3917 The remainder.

. . . .
22069810125 (2805
8

14069	Reſolvend.
6	The triple of 2
12	Triple ſquare of 2.
126	The diviſor.
512	The cube of 8.
384	Square of 8. by 6.
96	Triple Square by 8.
13952	The ſubtrahend.
117810125	New reſolvd.
84	Triple of 28.
2352	Triple ſqua. of 28
23604	The diviſor.
840	Triple of 280.
235200	Triple ſquare of 280.
2352840	New diviſor.
125	Cube of 5.
21000	Square of 5 by 840.
1176000	Triple ſquare by 5.
117810125	Subtrahend.

. . . .

In this *Example*, 13952, being ſubtracted from the reſolvend, 14069, the remainder is 117, to which bring down 810, the third point, & it makes 117810, for a new Reſolvend, and the next diviſor is 23604, which you cannot have in the ſaid reſolvend (the unit's place being rejected) ſo you muſt put 0 in the quotient, and ſeek a new diviſor; (after you have brought down your laſt point to the reſolvend) which new diviſor is 2352840, which you will find to be contained five times. So proceed to finiſh the reſt of the Work.

· · ·
25917056 (295.9
8

17917 The reſolvend.

6 The triple of 2.
12 The triple ſquare of 2.

126 The diviſor.

729 The cube of 9, the ſecond figure.
486 The ſquare of 9 by 6.
108 The triple ſquare by 9.

16389 The ſubtrahend.

1528056 The reſolvend.

87 The triple of 29.
2523 The triple ſquare of 29.

25317 The diviſor.

125 Cube of 5, the 3d fig.
2175 The ſquare of 5 by 87.
12615 The triple ſquare by 5.

1283375 The ſubtrahend.

244681000 The reſolvend.

885 The triple of 295.
261075 The triple ſquare of 295.

2611635 The diviſor.

729 The cube of 9, the laſt figure.
71685 The ſquare of 9 by 885.
2349675 The triple ſquare by 9.

235685079 The ſubtrahend.

8995921 The remainder.

In this *Example* I annex three ciphers to the remainder, which makes the 3d reſolvend; by which means I bring one Place of decimals. And ſo you may proceed to more decimal places at pleaſure, by annexing 3 ciphers to the next remainder, and carrying on the work as before.

· · · ·	
93759·575070 (45·42	
64	
29759	The reſolvend.
12	Triple of 4, the firſt figure.
48	Triple ſquare of 4.
492	diviſor.
125	Cube of 5, the ſecond figure.
300	The ſquare of 5, by 12, the triple quotient.
240	The triple ſquare by 5.
27125	The ſubtrahend.
2634575	The reſolvend.
135	The triple of 45.
6075	The triple ſquare of 45.
60885	The diviſor.
64	The cube of 4.
2160	The ſquare of 4 by 135.
24300	The triple ſquare by 4.
2451664	The ſubtrahend.
182911070	The Reſolvend
1362	The triple of 45.4.
618348	The triple ſquare of 45.4.
6184842	The diviſor.
8	The cube of 2.
5448	The ſquare of 2 by 1362.
1236696	The triple ſquare by 2.
123724088	The ſubtrahend.
59186982	The remainder.

In extracting the cube root of a mixed number, always obſerve to make the decimal part to conſiſt of either three, ſix, nine, *&c.* Places; that is, always to conſiſt of even points, as in the laſt *Example*, where the decimal places were five, to which I annexed a cipher, to make it up ſix, and ſo I proceed, to point it; and by that means I have a point falls upon the unit's place of whole numbers; which you muſt always obſerve.

To extract the Cube Root *out of a Fraction.*

This is the ſame to do as in whole Numbers, obſerve but the foregoing directions for the true pointing thereof; for, as was before directed, the decimal muſt always conſiſt of three, ſix, nine, *&c.* places; and if it be not ſo, it muſt be made ſo by annexing of ciphers, as is above ſaid.

If the cube root of a vulgar fraction be required, you muſt firſt reduce it to a decimal, and then extract the root thereof.

Examples of each follow.

Example 1. Let the cube root of .401719179 be required.

. . .

.401719179 (.737 root.
343 cube of 7.

58719 Reſolvend.

21 Triple of 7.
147 Triple ſquare of 7.

1491 Diviſor.

27 Cube of 3.
189 Square of 3 by 21.
441 Triple ſquare by 3.

46017 Subtrahend.

12702179 Reſolvend.

219 Triple of 73.
15987 Triple ſquare of 73.

160089 Diviſor.

343 Cube of 7.
10731 Square of 7 by 219.
111909 Triple ſquare by 7.

11298553 Subtrahend.

1403626 Remainder.

Example 2. Let the cube root of .001416 be required.

. . .
.000141600 (.052 Root.
125

16600 Reſolvend.

15 The triple of 5.
75 The triple ſquare of 5.

765 The diviſor.

8 The cube of 2.
60 The ſquare of 2 by 15.
150 The triple ſquare by 2.

15608 The ſubtrahend.

992 Remainder.

Example 3. Let $\frac{5}{276}$ be a vulgar fraction whoſe cube root is required.

By the firſt Rule of Chapter II. reduce the vulgar fraction to a decimal.

276) 5.000000000 (.018115942
....

2240
320
440
1640
2600
1160
560

8

.018115942

. . .

.018115942	(.262 Root.
8	
10115	Reſolvend.
6	The triple of 2.
12	The triple ſquare of .
126	The diviſor.
216	The cube of 6.
216	Square of 6 by the triple of 2.
72	The triple ſquare by 6.
9576	The ſubtrahend.
539942	Reſolvend.
78	The triple of 26.
2028	The triple ſquare of 26.
20358	The diviſor.
8	The cube of 2.
312	The ſquare of 2 by 78.
4056	Triple ſquare 2028 by 2.
408728	The ſubtrahend.
131214	Remainder.

You may prove the truth of the work by cubing the root found, as was ſhewn in the firſt *Example;* and if any thing remains, add it to the ſaid cube, and the ſum will be the given number, if the work is rightly performed.

I will ſhew the proof of the fifth *Example* (*Page* 48) the given number being 259697989, whoſe root is 638, it being a ſurd number there remains 3917.

	638
	638
	5104
	1914
	3828
The ſquare	407044
	638
	3256352
	1221132
	2442264
The cube	259694072
The remainder add	3917
Proof equal to the given number	259697989

EXTRACTION *of* ROOTS, *out of any Single Power, by one* General Method, *deduced from the* CONVERGING SERIES.

In order to perform this obſerve carefully the following.

GENERAL RULES.

Rule 1.] Point the given number as directed p. 32, 41, *&c.*

Rule 2.] Find the neareſt root to the firſt period towards the left hand, and to this root annex a cipher for every period, except one, in the given number, and raiſe it to the height of the given power.

Rule 3.] Take the difference between this involved root and this given number for a dividend, which *Note* with the ſign + if the given number be greater, but with — if the given number was leſs than the involved root.

Rule 4.] Raiſe the ſaid root and ciphers to a power whoſe index is one leſs than the index of the given power, and multiply this number by the index of the given power, for a diviſor, by which divide the dividend, taking but one figure into the quotient.

Rule 5.] If the aforeſaid difference was noted with +, this quotient figure muſt be annexed to the firſt taken root, but if noted with — it muſt be taken from the ſaid root, and the ſum or difference muſt be wrought with in every reſpect as with the firſt taken root, and the next quotient you get, you take two figures, and the next quotient after that you may take three or four figures, *&c.* every time nearly, doubling the number of figures, by which means you may have the root of any ſingle power to a great many places of figures, which compared with the old method is vaſtly eaſy: We ſhall examine this method by an *Example* or two of the *Square Root*, and then apply it to the Cube or higher Powers.

Example 4. What is the ſquare root of .321489?

OPERATION.

360000=Square of 600.
. . .
321489 given number ſubtract.

1200) 38511 (600 firſt root.
36000 30 quotient ſubtracted.

Remainder 251 560 ſecond root.

321489 given number.
313600 Square of 560 ſubtract.

1120)+7889 (560 ſecond root.
7840 7.0 Quotient add.

Remainder 49.0 567.0 third root.

Here the neareſt root to the firſt period 32 is 6, and there are two periods more in the given number, ſo I put two ciphers to 6 and it makes 600, which multiplied by itſelf gives 360000, which being greater than 321489, I ſubtract 321489 therefrom, and ſet the ſign — before the remainder or dividend 38511. Now the index of the ſquare being 2, and the next leſs power to the ſquare being 1, or the firſt power, (*Rule* 4.) the double of 600, *viz.* 1200, is the firſt diviſor, and the quotient comes out of 32, but we are to take but the firſt figure 3, ſo I put a cipher in the room of 2; and here *Note*, the reaſon of taking but one figure in the firſt diviſor, two figures in the ſecond diviſor, *&c.* is, becauſe if we take more, the root will be too much. Hence when the quotient is to be ſubtracted you may carry one to the firſt figure towards the right hand, ſo in this *Example*, inſtead of taking 30 from 600, I take 40 therefrom; for it is to be ſubtracted, becauſe 38511 is noted with —, the carrying of this 1, would ariſe from taking the 2 out of 0, *viz.* 32 from 600, but when the quotient is to be added, ſuch increaſe muſt not be regarded.

But 40 taken from 600 leaves 560 for the ſecond root, which gives 313600 when ſquared, and being leſs than the given number, is ſubtracted therefrom, and + 7889 remains for a ſecond dividend, which divided by twice 560, *viz.* 1120, gives 7.0; for here we are to take two figures into the quotient, and becauſe the dividend is *noted* with + this 7.0 is added to 560, which gives 567 for the third root, and being ſquared is equal to the given number; ſo 567 is the true ſquare root of 321489.

Example 5. What is the ſquare root of 7?

OPERATION.

9 the neareſt ſquare to 7.
7 the given number.

6)—2.0 ⎧ 3 the firſt root,
18 ⎩ 0.3 the quotient ſubtract,

Remainder (2) 2.6 ſecond root,

7 given number.
6.76 Square of the ſecond root,

5,2)+.240 (2. ſecond root.
208 .046 Quotient add,

320 2.646 third root,
312

(8) Remainder.

7.001316 Square of 2.646.
7——— given number ſubtract.
5.292)—.0013160 ⎧ 2.646 third root,
10584 ⎩ .0002486 Quot ſubtract

2576 2.6457513 fourth.
21168

⎧ Root true to the laſt
⎩ figure.

45920
42336

35840
31752

Remains (4088)

Here the neareſt ſquares to the given number 7, are 4 and 9, but 9 is nearer to 7 than 4, ſo I take 9, whoſe ſquare root is 3, and twice 3 is 6, the firſt diviſor; alſo becauſe the quotient 0.3 is to be ſubtracted from the firſt root 3, I ſubtract 0 4 as directed in the laſt *Example*; for the ſame reaſon I ſubtract 0002487, inſtead of .0002486 from the third root 2.646, and ſo get 2.6457513 for the ſquare root of 7, and if you ſquare this laſt root and proceed as before, you may rely on ſeven or eight figures in the quotient, and ſo have the root, to fourteen or fifteen places of decimals.

This method of extracting roots proves itſelf as it goes on, for here 3 is more than the ſquare root of 7, but 2.6 is ſomewhat leſs; alſo 2.646 is ſomewhat more, and again 2.6457513 is ſomewhat too little; ſo that every one of theſe roots come nearer the truth than the preceding one; that is, 2.6 is nearer than 3, but 2.646 is nearer than either 4, or 2 6, &c.

Example 6. What is the Cube Root of 1728?

. . OPERATION.

1728 the given number,
1000 the cube of 10 ſubtract.

300)+728 (10 firſt root,
600 2 quotient add,

Remains 128 12 ſecond root.

Here the cube root of 1, the firſt period, is 1, to which annex a cipher becauſe there is another period in the propoſed number 1728, and it becomes 10, whoſe cube 1000, taken from 1728 leaves 728 for a dividend; now the next power below the cube is the ſquare, and the index of the cube is 3, therefore by the fourth rule, the ſquare of 10 is 100, which multiplied by 3, gives 300 for the firſt diviſor, and the quotient 2 added to 10 gives 12 for the ſecond root, which cubed gives 1728; hence 12 is the exact cube root of 1728.

Example 7. Let it be required to extract the Cube Root out of

```
                          .067507824239 given num.
                            64           Cube of 4
                         ─────────────────
3 times the  ⎫ 48]+0035073242391 ⎫ .4 firſt root,
ſquare of .4=⎭        336         ⎭ .007 Q. add.
                      ───           ────
             Remains   14             .407 ſec. Rt.
                    ─────────────────
                    .067507824239 given numb.
                    .067419143 Sub.C.of. 407 2dR.
3 times    ⎫        ─────────────
the ſqu.   ⎬ .496947)+0000886812391 ⎫ .407      2d R.
of.047 is  ⎭          496947       ⎭ .000178 Q add
                      ───────        ────────
                     38986539  .407178(third
                      3478629  ⎧ root to the
                      ───────  ⎩ laſt figure.
                      4200249
                      3975576
                      ───────
             Remains  224673
```

NOTE, Becauſe the firſt figure that ariſes in the firſt quotient is 0, you may in ſuch caſes take two figures into the quotient, ſo here I take in 07, and becauſe two figures are taken into this quotient, you may take three or four Figures into the next quotient, as *per* Work.

o that the Cube Root of .067507824239 is .4; or nearer .407; or nearer .407178.

Example 8. What is the Cube Root of 282? And of 231?

OPERATION I.

343 Cube of 7.
282 given number,

147)—61.0 { 7 firſt root,
58.8 { 0.4 Quotient ſubtract,

Remains 22 6.5 ſecond root,

282 given Number,
274.625 Cube of 6.5

126.75)+7.3750 { 6.5 ſec. Root,
6.3375 { .058 Quotient add,

103750 6.558 third Root.
101400

Remains 2350

OPERATION II.

216
231

108)+150 { 6
108 { 0.1
Remains 42 6.1

231
226.981

111.63)+4.0190 { 6.1
33489 { .036
67010 6.136
66978

Remains 32

So the cube root of 282 is 6.558, And the Cube root of 231 is 61.36 nearly.

Example 9. What is the Cube Root of 2150.42 ?

OPERATION.

2150.42 given number,
1000 Cube of 10,

300)+1150.42 { 10 firſt root,
900 { 3 Quotient add,

Remains 250.42 13 ſecond root,

2197 Cube of 13,
2150.42 given number,

1521)—46.58 { 13 ſecond root.
4563 { 0.0306 Quot. ſubtr.

9500 12.9693 third root,
9126

Remains (374)

Example 10. What is the Cube Root of $1\frac{1}{3}$?

OPERATION.

The decimal of $1\frac{1}{3}$ is 1.3333 given number.
1 Cube of 1

3) 0.3333 { 1 firſt root.
·3 { 0.1 Quo. add

Remains .033 &c. 1.1 ſec. root.

1.3333 given number,
1.331 Cube of 1.1

3.63)+.002333 { 1. 1 ſecond Root,
2178 { .0006427.

1553 1.1006427
1452

1013
726

2873
2541

Remains (332)

In *Example* 9, Becauſe the firſt figure in the ſecond quotient is a cipher, I take four figures into that quotient, and ſo get the cube root of 2150.42 to be 12.9693, but in *Example* 10, the two firſt figures in the ſecond quotient are ciphers, *viz.* the places of hundreds and thouſands are each a cipher, and therefore in this caſe, I take figures into the ſaid quotient (you may take ſeven or eight figures if you pleaſe) and ſo get 1.1006327 for the Cube Root of $1\frac{1}{3}$, or 1.333333333, &c.

C H A P.

CHAP. IX.

Multiplication of Feet, Inches and Parts.

IN the multiplying of feet, inches, *&c.* I ſhall endeavour to lay down ſuch eaſy and familiar rules, as may eaſily be underſtood by the meaneſt capacity.

Example 1. Let 7 feet 9 inches be multiplied by 3 feet 6 inches.

F.	I.	
7	9	
3	6	
23	3	Prs.
3	10	6
27	1	6

Firſt, multiply 9 inches by 3, ſaying 3 times 9 is 27 inches, which make 2 feet 3 inches; ſet down 3 under inches, and carry 2 to the feet, ſaying, 3 times 7 is 21, and 2 that I carry make 23; ſet down 23 under the feet.

Then begin with 6 inches, ſaying 6 times 9 is 54 parts, which is 4 inches and 6 parts; ſet down 6 parts, and carry 4, ſaying, 6 times 7 is 42, and 4 that I carry is 46 inches, which is 3 feet 10 inches; which ſet down, and add all up together, and the product is 27 feet, 1 inch, 6 parts.

Example 2. Let 75 feet 7 inches be multiplied by 9 feet 8 inches.

F.	I.	
75	7	
9	8	
680	3	
50	4	8
730	7	8

Firſt, multiply by 9 feet, ſaying, 9 times 7 is 63, which is 5 feet 3 inches; ſet down 3 and carry 5, ſaying, 9 times 5 is 45, and 5 that I carry is 50; ſet down 0 and carry 5, ſaying, 9 times 7 is 63, and 5 is 68; ſet down 68, and proceed to multiply by 8 inches, ſaying, 8 times 7 is 56, the twelves in 56 are 4 times, and 8 remain; ſet 8 a place to the right hand, and carry 4; then multiply 75 by 8, and the product is 600, and 4 that I carry is 604, which divided by 12, the quotient is 50 feet, and 4 remains; ſet down 50 feet, 4 inches, and add all up together, and you will find the product 730 feet, 7 inches, 8 parts.

I will repeat the laſt *Example* again, and ſhew another way to work it, which I think is better and more expeditious when there are more figures than one in the feet: Thus,

F.	I.	P.	
75	7		
9	8		
680	3		
25	2	4	$\frac{1}{3}$
25	2	4	$\frac{1}{3}$
730	7	8	

Multiply by 9 feet firſt, as above directed; then, inſtead of multiplying by 8 inches, let the 8 inches be parted into ſuch aliquot or even parts of a foot as you find to be contained in that figure; if you take ſuch parts of the multiplicand, and add them to the former product, the ſum will give the anſwer, thus 8 inches may be parted into 4 and 4, becauſe 4 is the third part of 12: ſo, if you take the third part of 75 feet, 7 inches, and ſet it down twice, and add all together, the ſum will be 730 feet, 7 inches, 8 parts, the ſame as before: thus ſay how often 3 in 7, which is twice, ſet down 2; then, becauſe twice 3 is 6, ſay 6 out of 7 and there remains 1, for which you muſt add 10 to the 5, and it makes 15; then the threes in 15 are 5 times, ſet down 5; and, becauſe 3 times 5 is 15, there is 0 remains. Then go to the 7 inches; ſaying the threes in 7 are twice, ſet down 2 in the inches;

becauſe twice 3 is 6, take 6 out of 7 and there remains 1 inch, which is 12 parts; then the threes in 12 are 4 times, and 0 remains. So the third part of 75 feet, 7 inches, is 25 feet, 2 inches, 4 parts; which ſet down again, and add all together, the ſum is 730 feet, 7 inches, 8 parts, the ſame as before.

Example 3. Let 97 feet, 8 inches, be multiplied by 8 feet, 9 inches.

F.	I.	
97	8	$\frac{1}{2}$
8	9	$\frac{1}{4}$
781	4	
48	10	
24	5	
854	7	

Begin, firſt to multiply by 8 feet, ſaying, 8 times 8 is 64 inches, that is, 5 feet 4 inches; ſet down 4 inches, and carry 5, ſaying, 8 times 7 is 56, and 5 I carry is 61; ſet down 1, and carry 6, ſaying, 8 times 9 is 72, and 6 I carry is 78, which ſet down: then inſtead of multiplying by 9 inches, take the aliquot parts of 12, which makes 9, which is 6 and 3, 6 inches being half 12, and 3 the fourth part; therefore take the half of 97 feet, 8 inches, which is 48 feet, 10 inches, and becauſe 3 is half 6, you may take the half of 48 feet, 10 inches, which is 24 feet 5 inches; add all up together, and the ſum is 854 feet, 7 inches. See the work, as above.

Example 4. Let 75 feet, 9 inches, be multiplied by 17 feet, 7 inches.

F.	I.	P.
75	9	
17	7	
525		
75		
25	3	
18	11	3
8	6	
4	3	
1331	11	3

In this *Example*, becauſe there are more than 12 feet in the multiplier, therefore I firſt multiply the 75 by 17 feet: then becauſe the aliquot parts in 7 inches are 4 and 3, that is, a third and a fourth I take the third part of 75 feet, 9 inches, which is 25 feet, 3 inches, and the fourth part thereof is 18 feet, 11 inches, 3 parts; then the aliquot parts of 9 inches are 6 and 3, that is half and a fourth; therefore I take half 17 feet which is 8 feet, 6 inches, and the fourth part is 4 feet 3 inches, not meddling with the 7 inches, becauſe that was multiplied into the 9 before, then add all theſe together, and the ſum is 1331 feet, 11 inches, 3 parts.

Example 5. Let 87 feet, 5 inches, be multiplied by 35 feet, 8 inches.

F.	I.	P.	
87	5		
35	8		
435			
261			
29	1	8	$\frac{1}{3}$
29	1	8	$\frac{1}{3}$
11	8	0	$\frac{1}{3}$
2	11	0	$\frac{1}{12}$
3117	10	4	

Work here as in the laſt *Example*. After you have multiplied the feet, then take the aliquot parts of 8 inches, which is two thirds; therefore take the third part of 87 feet, 5 inches, and ſet it down twice; thus the third part of 87 feet, 5 inches is 29 feet 1 inch 8 parts; ſet this down twice: then the aliquot parts of 5 inches are 4 and 1, that is, a third part and a twelfth part; therefore take a third part of 45, which is 11 feet, 8 inches, and a twelfth part of 35 is 2 feet, 11 inches, ſet all theſe one under another, and add them together, and the ſum is 3117 feet, 10 inches, 4 parts.

Example 6. Let 259 feet, 2 inches, be multiplied by 48 feet, 11 inches.

F.	I.	P.		
259	2			
48	11			
2072				
1036				
129	7		½	
86	4	8	⅓	
21	7	2	½	F. I.
8	1	10	⅙	of 48 11
12677	8	8		

Firſt, multiply the feet; then take the aliquot parts of 11, which will be 6, 4 and 1, that is, an half a third, and a twelfth; therefore take the half of 259 feet, 2 inches, which is 129 feet 7 inches, and a third part is 86 feet 4 inches 8 parts, and the twelfth part of 259 feet, 2 inches, is 21 feet, 7 inches, 2 parts; or (becauſe 1 is the fourth part of 4) you may more readily take the fourth part of 86 feet, 4 inches, 8 parts, which is alſo 21 feet, 7 inches, 2 parts; then becauſe 2 inches (in the multiplicand) is one ſixth of 12, take the ſixth part of the feet, only, in the multiplier, *viz*. 48, which quotes 8 feet, and add all together, and the sum is 12677 feet, 6 inches, 10 parts. See the foregoing work corrected.

I ſhall ſet down only the working of ſome few *Examples* in feet and inches, and then proceed to multiply feet, inches, and parts, *&c.*

F.	I.	P.		
179	3			
38	10			
1432				
537				
89	7	6	½	
59	9	0	⅓	F.
9	6	0	¼	*of* 38
Pro. 6960	10	6		

F.	I.	P.		
246	7			
46	4			
1476				
984				
82	2	4	⅓	F.
15	4	0	⅓	*of* 46
11	6	0	¼	
Prod. 11425	0	4		

	F.	I.	P.		
	246	7			
	36	9			
	1476				
	738				
	123	3	6	½	
	61	7	9	¼	F.
	12	0	0	⅓	} of 36
	9	0	0	¼	
Prod.	9061	11	3		

	F.	I.	P.		
	257	9			
	39	11			
	2313				
	771				
	128	10	6	½	
	85	11	0	⅓	
	21	5	9	1/12	F.
	19	6	0	½	} of 39
	9	9	0	¼	
Prod.	10288	6	3		

Example 11. Let 7 feet, 5 inches, 9 parts be multiplied by 3 feet, 5 inches, 3 parts.

F.	I.	P.	S.	T.
7	5	9		
3	5	3		
22	5	3		
3	1	4	9	
	1	10	5	3
25	8	6	2	3

In this *Example*, I firſt begin with 3 feet, and thereby multiply 7 feet 5 inches 9 parts: firſt I ſay, 3 times 9 is 27 parts, that this, 2 inches and 3 parts, ſet down 3 under the parts, and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 foot 5 inches; ſet down 5 inches and carry 1, and ſay, 3 times 7 is 21, and 1 I carry is 22; ſet down 22 feet. Then begin with 5 inches, ſaying, 5 times 9 is 45, which is 45 ſeconds, which makes 3 parts and 9 ſeconds; ſet down 9 ſeconds a place towards the right hand, and carry 3 parts, ſaying, 5 times 5 is 25 and 3 I carry is 28, which is 2 inches and 4 parts; ſet down 4 parts, and carry 2, ſaying, 5 times 7 is 35 and 2 I carry is 37, which is 3 feet 1 inch; ſet down 3 feet 1 inch, and begin to multiply by 3 parts, ſaying, 3 times 9 is 27 thirds, that is, 2 ſeconds and 3 thirds;

ſet down 3 thirds and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 part and 5 ſeconds; ſet down 5 ſeconds, and carry 1, ſaying, 3 times 7 is 21, and 1 I carry is 22, which is 1 inch and 10 parts; which ſet down, and add all up, and the product is 25 feet, 8 inches, 6 parts, 2 ſeconds, 3 thirds.

NOTE, That in multiplying feet inches and parts, *&c.* if feet be multiplied by feet, the product is feet; and feet multiplied by inches, the product is inches, and the twelfth part is feet; and parts multiplied by feet, the product is parts, and the twelfth part thereof is inches; parts multiplied by inches, the product is ſeconds, and the twelfth part thereof is parts; and parts multiplied by parts, the product is thirds, and the twelfth part thereof is ſeconds.

So, that if you begin to multiply parts by feet in the firſt row, and parts by inches in the ſecond row, and parts by parts in the third row, the firſt figure in every row will ſtand a place more towards the right hand, as you may ſee in the laſt *example*.

Example 12. Let 37 feet, 7 inches, 5 parts, be multiplied by 4 feet, 8 inches, 6 parts.

F.	I.	P.			
37	7	5			
4	8	6			
150	5	8	S.		
12	6	5	8		$\frac{1}{3}$
12	6	5	8	T.	$\frac{1}{3}$
1	6	9	8	6	$\frac{1}{2}$
177	1	5	0	6	

Firſt, I multiply by 4 feet, ſaying, 4 times 5 is 20, which is 1 inch 8 parts; ſet down 8, and carry 1, ſaying 4 times 7 is 28, and 1 I carry is 29, which is 2 feet, 5 inches; ſet down 5 inches, and carry 2, ſaying, 4 times 7 is 28 and 2 I carry is 30; ſet down 0, and carry 3, and ſay 4 times 3 is 12 and 3 is 15; ſet down 15. Then I begin with 8 inches; but becauſe the feet in the multiplicand are more than 12, it will be the beſt way to work for the aliquot parts of 8; ſo here I work for 4 inches, and ſet that down twice, 4 being the third part of 12; therefore take the third part of 37 feet, 7 inches, 5 parts, which is 12 feet, 6 inches, 5 parts, 8 ſeconds; ſet this down twice. Then begin with 6 parts; but, inſtead of multiplying, take half 37 feet, 7 inches, 5 parts; (becauſe 6 is half 12) and ſet it a place more to the right hand. Thus, the half of 37 feet is 18, which I muſt count 18 inches, becauſe the multiplier is 6 parts; ſo the half of 37 feet, 7 inches, 5 parts, is 1 foot, 6 inches, 9 parts, 8 ſeconds, 6 thirds; which ſet down, and add all up together, and the ſum is 177 feet, 1 inch, 5 parts, 0 ſeconds, 6 thirds.

Example 13. Let 311 feet, 4 inches, 7 parts, be multiplied by 36 feet, 7 inches, 5 parts.

F.	I.	P.	S.	T.	
311	4	7			
36	7	5			
1866					
933					
103	9	6	4		$\frac{1}{3}$
77	10	1	9		$\frac{1}{4}$
8	7	9	6	4	$\frac{1}{3}$
2	1	11	4	7	$\frac{1}{4}$
12	0	0	0	0	$\frac{1}{3}$
1	0	0	0	0	$\frac{1}{12}$
	9	0	0	0	$\frac{1}{3}$
11402	2	4	11	11	

In this *Example*, becauſe the feet both in the multiplier and multiplicand are compound numbers, I firſt multiply the feet one by the other; then take the aliquot parts of 7 inches, which are 4 inches and 3, that is, a third and a fourth part; ſo take the third part of 311 feet, 4 inches, 7 parts, which is 103 feet, 9 inches, 6 parts, 4 ſeconds, and the fourth part is 77 feet, 10 inches, 1 part, 9 ſeconds; ſet theſe down one under another, the feet under the other feet; then the aliquot parts of 5 parts are 4 and 1, that is, a third, and twelfth part; ſo the third part of 311 feet, 4 inches, 7 parts is 103 feet, 9 inches, 6 parts, 4 ſeconds; but becauſe the multiplier is parts, it muſt be ſet a place to the right hand, that is, the 103 muſt be inches, which is 8 feet, 7 inches; therefore I ſet down 8 feet, 7 inches, 7 parts, 6 ſeconds, 4 thirds.

Then, becauſe 1 inch is a fourth part of 4 inches, therefore I take a fourth part of 8 feet, 7 inches, 9 parts, 6 ſeconds, 4 thirds, which is 2 feet, 1 inch, 11 parts 4 ſeconds, 7 thirds; which is the ſame as if I had taken a twelfth part of 311 feet, 4 inches, 7 parts, Then, for 4 inches in the multiplicand, inſtead of multiplying 36 feet by it, take a third part, becauſe 4 inches is a third part of 12; ſo the third part of 36 is 12 feet, and the aliquot parts of 7 parts are 4 and 3, that is a third and a fourth; ſo the third part of 36 is 12, which now is 12 inches, that is, 1 foot, and the fourth part is 9 inches; add all theſe together, and the ſum will be 11402 feet, 2 inches, 4 parts, 11 ſeconds, 11 thirds.

Example 14. Let 8 feet, 4 inches, 3 parts, 5 ſeconds, 6 thirds, be multiplied by 3 feet, 3 inches, 7 parts, 8 ſeconds, 2 thirds.

	F.	I.	P.	S.	T.				
	8	4	3	5	6				
	3	3	7	8	2				
	25	0	10	4	6				
	2	1	0	10	4	6			
		4	10	6	0	2	6		
			5	6	10	3	8	0	
				1	4	8	6	11	0
Product	27	7	3	5	1	8	8	11	0

In this laſt *Example* there is no difficulty, if you do but obſerve the former directions, and ſet every row a place more to the right hand, I ſhall only ſet down the working of ſome few *Examples* more, and ſo conclude this chapter,

F.	I.	P.		
321	7	3		
9	3	6		
2894	5	3	S.	
80	4	9	9	T.
13	4	9	7	6
2988	2	10	4	6

F.	I.	P.	
42	7	8	
7	3	6	
298	5	8	
10	7	11	S.
1	9	3	10
310	10	10	10

F.	I.	P.		
124	7	9		
14	6	2		
496				
124			S.	
62	3	10	6	T.
1	8	9	3	6
7	0	0	0	0
1	2	0	0	0
	7	0	0	0
	3	6	0	0
1809	1	1	9	6

F	I.	P.		
259	10	8		
18	5	4		
2072				
259			S.	
86	7	6	8	
21	7	10	8	T.
7	2	7	6	8
9	0	0	0	0
6	0	0	0	0
1	0	0	0	0
4793	6	0	10	8

F.	I.	P.		
267	7	10		
25	9	7		
1335				
534				
133	9	11	S.	
66	10	11	6	
11	1	9	11	T.
1	10	3	7	10
12	6	0	0	0
2	1	0	0	0
1	0	6	0	0
	8	1	0	0
6905	0	7	0	10

F.	I.	P.		
317	9	7		
37	5	9		
2219				
951			S.	
105	11	2	4	
26	5	9	7	T.
13	2	10	9	6
6	7	5	4	9
18	6	0	0	0
9	3	0	0	0
1	6	6	0	0
	3	1	0	0
11910	9	11	1	3

THE RULE AT LARGE.

$$\text{\textit{Feet multiplied by}} \left\{ \begin{array}{c} \text{\textit{feet}} \\ \text{\textit{inches}} \\ \text{\textit{parts}} \end{array} \right\} \text{\textit{produce}} \left\{ \begin{array}{l} \text{\textit{feet.}} \\ \text{\textit{inches.}} \\ \text{\textit{parts.}} \end{array} \right.$$

$$\text{\textit{Inches multiplied by}} \left\{ \begin{array}{c} \text{\textit{feet}} \\ \text{\textit{inches}} \\ \text{\textit{parts}} \end{array} \right\} \text{\textit{produce}} \left\{ \begin{array}{l} \text{\textit{inches.}} \\ \text{\textit{parts.}} \\ \text{\textit{ſeconds.}} \end{array} \right.$$

$$\text{\textit{Parts multiplied by}} \left\{ \begin{array}{c} \text{\textit{feet}} \\ \text{\textit{inches}} \\ \text{\textit{parts}} \end{array} \right\} \text{\textit{produce}} \left\{ \begin{array}{l} \text{\textit{parts.}} \\ \text{\textit{ſeconds.}} \\ \text{\textit{thirds.}} \end{array} \right.$$

N. B. ***Twelve thirds make one ſecond, twelve ſeconds make one part, twelve parts make one inch, and twelve inches make one foot, in this multiplication of feet, inches and parts.***

THE

Complete Meaſurer.

PART II.

CHAP. I.

Menſuration of SUPERFICIES.

Superficial figures are all ſuch as have only length and breadth, not having any commenſurable thickneſs.

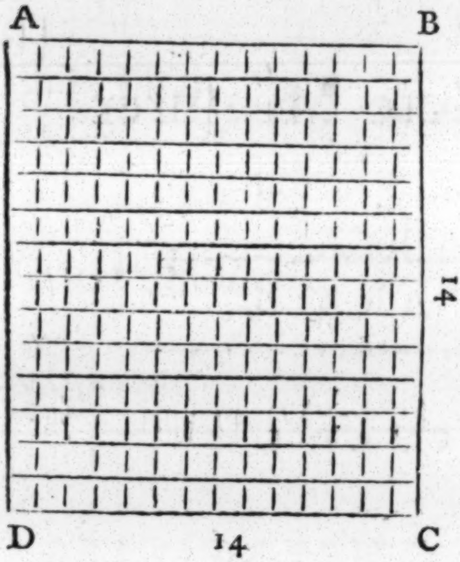
§. 1. *Of a* SQUARE.

A SQUARE is a geometrical figure; having four equal ſides, and as many right (or ſquare) angles. To find the ſuperficial content thereof, this is

The RULE.

Multiply the ſide into itſelf, and the product is the content.

Let ABCD be a geometrical ſquare given, each ſide being 14 feet, yards, poles, or other meaſure; multiply 14 by itſelf, and the product is 196, which is the ſuperficial content.



14
14
56
14
196 Product.

By Scale *and* Compaſſes.

Extend the compaſſes from 1, in the line of numbers, to 14; the ſame extent will reach from the ſame point turned forward to 196.

DEMONSTRATION.

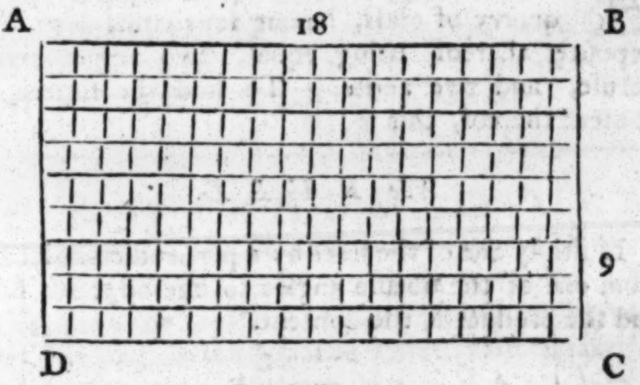
Let each ſide of the given ſquare be divided into 14 equal parts, and lines drawn from one another, croſſing each other within the ſquare; ſo ſhall the whole great ſquare be divided into 196 little ſquares, as you may ſee in the figure above, equal to the number of ſquare feet, yards, or poles, or other meaſures, by which the ſide was meaſured.

§ II. *Of a* PARALLELOGRAM, *or* LONG-SQUARE.

A PARALLELOGRAM is a figure having four ſides, and as many right angles, the oppoſite ſides thereof being equal and parallel. To find the ſuperficial content thereof this is

The RULE.

Multiply the length by the breadth, and the product is the ſuperficial content.



Length 18
Breadth 9

Product 162

Let ABCD be a long ſquare, the length thereof 18 feet, and the breadth 9 feet, which multiplied together, the product is 162, the ſuperficial content thereof.

By Scale *and* Compaſſes.

Extend the compaſſes in the line of numbers from 1 to 9, the ſame extent will reach from 18 down to 16 the ſquare feet.

DEMONSTRATION.

DEMONSTRATION.

If the ſides AB and CD, be each divided into 18 equal parts, repreſenting 18 feet, and the lines AD and BC each divided in 9 equal parts, and lines drawn from point to point, croſſing each other within the figure; thoſe lines will make thereby ſo many little ſquares as there are ſquare feet, *viz.* 162.

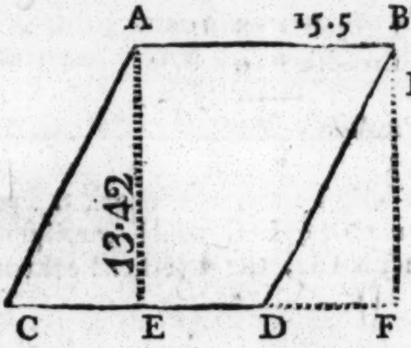
---

§ III. *Of a* RHOMBUS.

A RHOMBUS is a figure repreſenting a quarry of glaſs, having four equal ſides, the oppoſites thereof being equal, two angles being obtuſe, and two acute. To find the ſuperficial content thereof, this is

The RULE.

Multiply one of the ſides by a perpendicular let fall from one of the obtuſe angles to the oppoſite ſide, and the product is the content.



Perpendicular 13.42
The ſide 15.5

6710
6710
1342

Product 208.010

Let ABCD be a rhombus given whoſe ſides are each 15.5 feet, and the perpendicular EA is 13.42, which is the ſuperficial content of the rhombus that is, 208 feet, and one hundredth part of a foot.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 13.42; that extent will reach from 15.5, the ſame way to 208 feet, the content.

DEMONSTRATION.

Let CD be extended out to F. making DF equal to CE, and draw the line BF; ſo ſhall the triangle DBF be equal to the triangle ACE: For DF and CE are equal, and BF is equal to AE, becauſe AB and CF are parallel. Therefore the parallelogram ABEF is equal to the rhombus ABCD.

---

§ IV. *Of a* RHOMBOIDES.

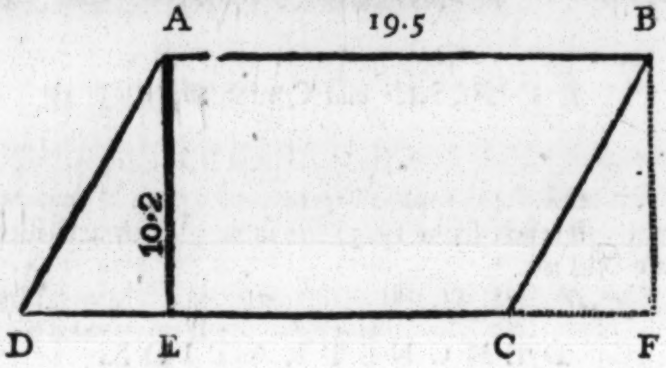
A RHOMBOIDES is a figure having four ſides, the oppoſite whereof are equal and parallel; and alſo four angles, the oppoſite whereof are equal. To find the ſuperficial content thereof, this is

The RULE.

Multiply one of the longeſt ſides thereof by the perpendicular let fall from one of the obtuſe angles to one of the longeſt ſides, and the product is the content.

```
  19.5
  10.2
 -----
   390
 1950
 -----
 198.90
```

Let



Let ABCD be a rhomboides given whoſe longeſt ſide AB or CD is 19.5 feet, and the perpendicular AE is 10.2; which multiplied together, the product is 198.9, that is, 198 ſuperficial feet, and 9 tenth parts, the content.

DEMONSTRATION.

If DC be extended to F, making CF equal to DE, and a line drawn from B to F; ſo will the triangle CBF be equal to the triangle ADE, and the parallelogram AEFB be equal to the rhomboides ABCD. Which was to be proved.

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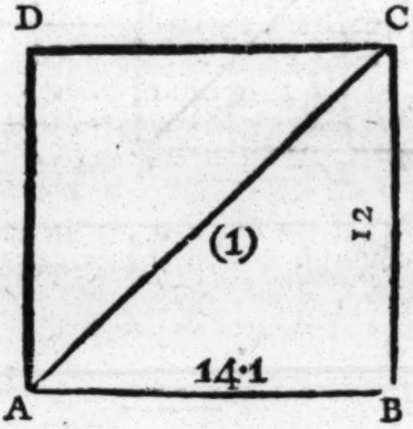
§ V. *Of a* TRIANGLE.

A TRIANGLE is a figure having three ſides and three angles. Triangles are right angled or oblique angled. Right angled triangles are ſuch as have one right angle. Oblique angled triangles are ſuch as have their angles either acute or obtuſe. An obtuſe angle is greater than a right angle, that is, it is more than 90 degrees; and an acute angle is leſs than a right angle. To find the ſuperficial content thereof, this is

The

The RULE.

Let the triangle be of what kind ſoever, multiply the baſe by half the perpendicular, or half the baſe by the whole perpendicular; or multiply the whole baſe by the whole perpendicular, and take half the product: any of theſe three ways will give the content.



Let ABC be a right-angled triangle whoſe baſe is 14.1 feet, and the perpendicular 12 feet, multiply 14.1 by 6, half the perpendicular, and the product is 84.6 feet, the content. Or multiply 14.1 by 12, the product is 169.2; the half thereof is 84.6. the ſame as before.

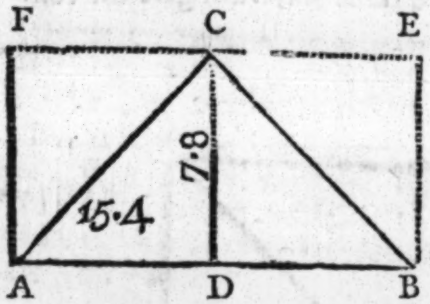
14.1 Baſe. .	14.1 Baſe.
6 Half perpendicular.	12 Perpendicular.
84.6 Product.	169.2 Product.
	84.6

By

By Scale *and* Compaſſes.

Extend the compaſſes from 2 to 14.1, that extent will reach the ſame way from 12 to 84.6 feet, the content.

Fig. 2.



15.4	Baſe,
3.9	half perpen.
1386	
462	
60.06	Product.

15.4	Baſe	7.7	Half Baſe.
7.8	Perpend.	7.8	Perpend.
1232		616	
1078		39	
120.12		60.06	Product.
60.06			

Let ABC (*fig.* 2) be an oblique-angled triangle given, whoſe baſe is 15.4, and the perpendicular 7.8; if 15.4 be multiplied by 3.9, (half the perpendicular) the product will be 60.06 for the area or ſuperficial content: or, if the perpendicular 7.8 be multiplied into half the baſe 7.7 the product will be 60.06, as before: or, if 15.4, the baſe, be multiplied by the whole perpendicular 7.8, the product will be 120.12, which is the double area; the half thereof is 60.06 feet, as before.

See the Work.

By Scale *and* Compaſſes.

Extend the compaſſes from 2 to 15.4, that extent will reach from 7.8 to 60 feet, the content.

DEMONSTRATION.

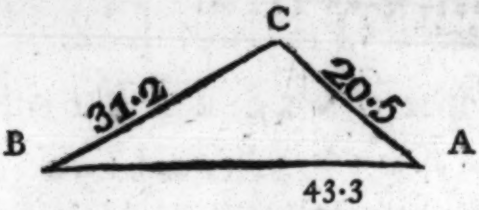
If AD. (*Fig.* 1.) be drawn parallel to BC, and DC parallel to AB; the triangle ADC ſhall be equal to the given triangle ABC. Hence the parallelogram ABCD. is double to the given triangle; therefore half the area of the parallelogram is the area of the triangle. In *Fig.* 2. the parallelogram ABEF is alſo double to the triangle ABC; for the triangle ACF is equal to the triangle ACD, and the triangle BCE is equal to the triangle BCD; therefore the area of the parallelogram is double to the area of the given triangle. Which was to be proved.

To find the area of any plain triangle, by having the three ſides given, without the help of a perpendicular.

RULE.

Add the three ſides together, and take half that ſum; then ſubtract each ſide ſeverally from that half ſum. Which done, multiply that half ſum and the three differences continually, and out of the laſt product extract the ſquare root, which ſquare root ſhall be the area of the triangle ſought.

Example. Let ABC be a triangle, whoſe three ſides are as follow, *viz.* AB 43.3, AC 20.5, and BC 31.2; the area is required.



$$\text{Sides}\left\{\begin{array}{r}43.3\\31.2\\20.5\end{array}\right\}\left\{\begin{array}{r}4.2\\16.3\\27.0\end{array}\right\}\text{Differences.}$$

Sum 95.0

Half 47.5

Area 296.31 47.5 The half ſum.
27 Difference.

3325
950

1282.5 Product.
16.3 Difference.

38475
76950
12825

20904.75 Product.
4.2 Difference.

4180950
8361900

. . . . .
87799.9500 (296.31
4

49) 477
441

586) 3699
3516

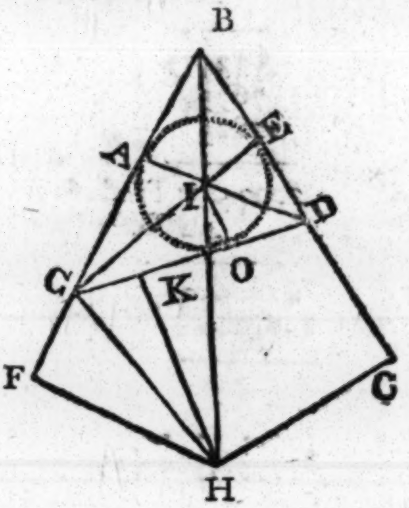
5923) 18395
17769

59261) 62600
59261

3339 Remains.

DEMONSTRATION.

In the triangle BCD, I ſay, if from the half ſum of the ſides you ſubtract each particular ſide, and multiply the half ſum and the three differences together continually, the ſquare root of the product ſhall be the area of the triangle.



Firſt, by the lines BI, CI and DI, biſect the three angles, which lines will all meet in the point I; by which lines the given triangle is divided into three new triangles CBI, DCI, and BDI; the perpendiculars of which new triangles are the lines AI, EI and OI, being all equal to one another, becauſe the point I is the center of the inſcribed circle (by *Euclid*, *Lib.* 4 *Prop.* 4) Wherefore to the ſide BC join CF equal to DE or DO; ſo ſhall BF be equal to half the ſum of the ſides, *viz.*= $\frac{1}{2}$ BC + $\frac{1}{2}$ BD + $\frac{1}{2}$ CD.

And BA=BF—CD; for CA=CO and OD=CF, therefore CD=AF, and AC=BF—BD; for BE=BA and ED=CF; therefore BD=BA+CF, and CF=BF—BC.

Then make CK=CE, and draw the perpendiculars FH, GH and KH, and extend BI to H; becauſe the angles FCK+FHK are equal to two right angles, (for the angles F and K are right angles) equal alſo to FCK+ACO (by *Euclid*. 1. 13.) And the angles ACO+AIO are equal to two right angles; therefore the quadrangles FCKH and AIOC are alike; and the triangles CFH and AIC are alſo ſimilar. And the triangles BAI and BFH are likewiſe ſimilar.

From this explanation, I ſay the ſquare of the area of the given triangle will be BF *q* × IA*q*=BF × BA × CA × CF: In words:

The ſquare of BF (the half ſum of the ſides) multiplied into the ſquare of IA (IA=IF=IO) will be equal to the half ſum multiplied into all the three differences,

For IA : BA : : FH : BF, and IA : CF : : AC : FH; becauſe the triangles are ſimilar, by *Euclid Lib.* 6 *Prop.* 4.

Wherefore multiplying the extreams and means in both, it will be IA *q* × BF × FH=BA × CA × CF × FH; but FH being on both ſides of the equation, it may be rejected; and then multiply each part by BF, it will be BF *q* × IA *q*=BF × BA × CA × CF, Which was to be demonſtrated.

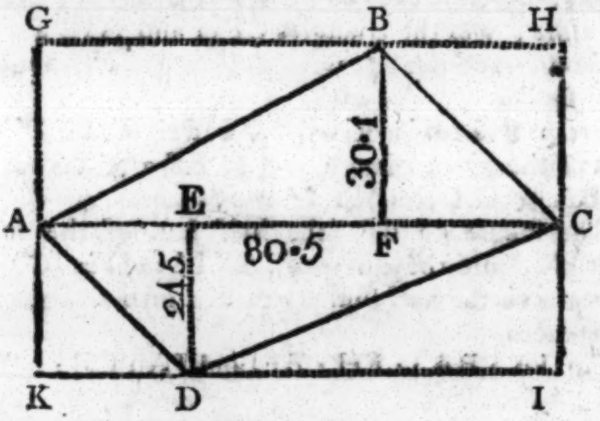
§ VI. *Of a* TRAPEZIUM.

A TRAPEZIUM is a figure having four unequal ſides and oblique angles. To find the area or ſuperficial content thereof, this is

The RULE.

Add the two perpendiculars together, and take half the ſum, and multiply that half ſum by the diagonal, or multiply the whole ſum by half the diagonal, the product is the area. Or you may find the areas of the two triangles ABC and ACD, (by ſection V.) and add thoſe two areas together, the ſum ſhall be the area of the trapezium.

Or, *laſtly*, multiply the whole diagonal by the ſum of the perpendiculars, and take half the product for the area of the trapezium.



BF=30.1
DE=24.5

Sum 54.6

Half 27.3
AC=80.5

1365
2184

Area 2197.56

Let ABCD be a trapezium given, the diagonal whereof is 80.5 and the perpendicular BF 30.1, and the perpendicular DE 24.5; theſe two added together the ſum is 54.6, the half thereof is 27.3, which multiplied by the diagonal 80.5 the product is 2197.65, which is the area of the trapezium; or if 4025, half the diagonal, be multiplied by 54.6, the whole ſum of the perpendiculars, the product is 2197.65, the ſame as before.

By

By Scale *and* Compaſſes.

Extend the compaſſes from 2 to 54.6; that extent will reach from 80.5 to 2197.65, the area.

DEMONSTRATION.

This figure, ABCD, is compoſed of two triangles, the triangle ABC, is half the parallelogram AGHC: Alſo the triangle ACD is equal to half the parallelogram ACIK, as was proved § V. Wherefore the trapezium ABCD is equal to half the parallelogram GHIK. To find the area $HI = BF + DE$, therefore $\frac{1}{2} HI \times AC (= KI = GH) =$ area of the trapezium. Which was to be proved.

§ VIII. *Of* IRREGULAR FIGURES.

IRREGULAR figures are all ſuch as have more ſides than four, and the ſides and angles unequal. All ſuch figures may be divided into as many triangles as there are ſides, wanting two. To find the area of ſuch figures, they muſt be divided into trapeziums and triangles, by lines drawn from one angle to another, and ſo find the areas of the trapeziums and triangles ſeverally, and then add all the areas together, ſo will you have the area of the whole figure.

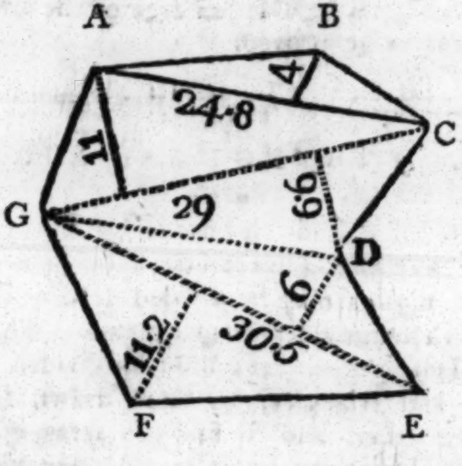
Let ABCDEFG be an irregular figure given to be meaſured; firſt, draw the lines AC and GD, and thereby divide the given figure into two trapeziums, ACGD and GDEF and the triangle ABC; of all which I find the areas ſeverally.

Firſt, I multiply the baſe AC by half the perpendicular, and the product is 49.6, the area of the triangle ABC.

Then, for the trapezium, ACGD, the two perpendiculars 11 and 6.6 added together make 17.6; the half thereof is 8.8 which multiplied by 29, the diagonal, the product is 255.2 the area of that trapezium.

And for the trapezium GDEF, the two perpendiculars, 11.2 and 6, added together make 17.2 the half thereof is 8.6, which multiplied by 30.5 the diagonal, the product is 262.3 the area thereof. All theſe areas added together make 567.1, and ſo much is the area of the whole irregular figure.

See the work.



24.8 Baſe AC.
2 half perpendic.

49.6 Area of ABC

11 } Perpendiculars.
6.6 }

17.6 Sum.

8.8 Half.
29 diag. CG.

792
176

255.2 Area of ACGD.

11.2	Perpendiculars.	30.5	
6.		8.6	
17.2	Sum.	1830	
8.6	Half ſum.	2440	
		262.30	Area of GDEF,
		255.2	Area of ACGD,
		49.6	Area of ABC.
		567.1	Sum of the areas.

The figure being compoſed of triangles and trapeziums, and thoſe figures being ſufficiently demonſtrated in the fifth and ſixth ſections aforegoing, it will be needleſs to mention any thing of the demonſtration thereof in this place.

§ VIII. *Of* Regular Polygons.

REGULAR polygons are all ſuch figures as have more than four ſides, all the ſides and angles thereof being equal polygons, are denominated from the number of their ſides and angles.

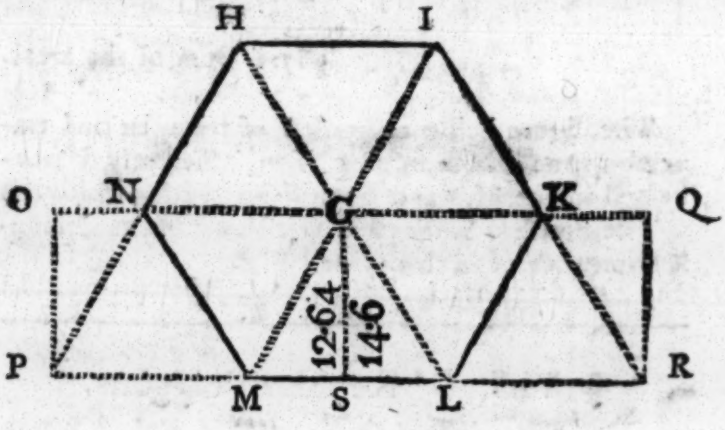
If the figure conſiſts of	5	Equal ſides and angles it is called a regular	Pentagon
	6		Hexagon
	7		Heptag.
	8		Octagon
	9		Enneag.
	10		Decagon
	11		Endecag.
	12		Dodecag.

To find the area or ſuperficial content of any regular polygon, this is

The

The R U L E.

Multiply the whole perimeter, or ſum of the ſides, by half the perpendicular, let fall from the center to the middle of one of the ſides ; or multiply the half perimeter by the whole perpendicular, and the product is the area.



14.6
3

43.8 Half ſum of the ſides.

12.64 The perpend.
43.8 Half ſum.

10112
3792
5056

553.632 Area

14.6
6

87.6 Sum of the ſides,
6.32 Perpend.

1752
2628
5256

553.632 Area.

Let HIKLMN be a regular hexagon, each ſide thereof being 14.6, the ſum of all the ſides is 87.6, the half ſum thereof is 43.8, which multiplied by the perpendicular GS 12.64, the product is 553.632: Or if 87.6, the whole ſum of the ſides, be multiplied by half the perpendicular 6.32, the product is 553.632; the ſame as before; which is the area of the given hexagon.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 12.6, that extent will reach from 43.8, the ſame way to 553.63: Or extend from 2 to 12.6, that extent will reach from 87.6 to 553.63.

DEMONSTRATION.

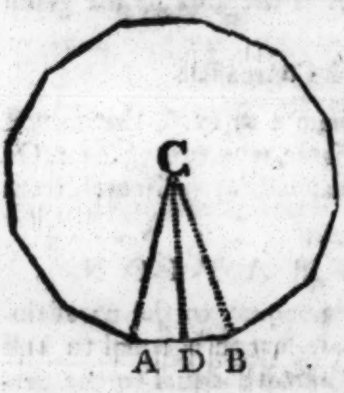
Every regular polygon is equal to the parallelogram, or long ſquare whoſe length is equal to half the ſum of the ſides, and breadth equal to the perpendicular of the polygon as appears by the foregoing figure; for the hexagon HIKLMN is made up of ſix equilateral triangles: And the parallelogram OPQR is alſo compoſed of ſix equal and equilateral triangles, that is, five whole ones, and two half ones; therefore the parallelogram is equal to the hexagon.

A TABLE *for the more ready finding the area of a* POLYGON.

Number of ſides.	Names.	Multiplier.
3	Trigon	.433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3.633931
8	Octagon	4.828424
9	Enneagon	6.181821
10	Decagon	7.694205
11	Endecagon	9.365653
12	Dodecagon	11.196150

Multiply the ſquare of the ſide by the tabular number: and the product is the area of the polygon.

How to find theſe tabular numbers.



Theſe numbers are found by trigonometry, thus: find the angle at the center of the polygon by dividing 360 degrees by the number of ſides of the polygon.

Example. Suppoſe each ſide of the dodecagon annexed be 1, and the area be required.

Divide, 360 by 12, (the number of ſides) the quotient is 30 degrees for the angle ABC; the half thereof is 15, the angle DCB, whoſe complement to 90 degrees is 75 degrees, the angle CBD: Then ſay,

As s. DCB 15 degrees,—C. Ar. 0.587004
To 5, the half ſide DB Log.—1.698970
So is s. CBD 75 degrees - - - - 9.984944

To the perpendicular CD 1.866025 2.270918

Then 1.866025 multiplied by .5 (the half ſide) the product is .9330125, = the area of the triangle ACB, which again multiplied by 12, the number of ſides in the dodecagon, produces 11.196150 * the area of the dodecagon required whoſe ſide is = 1.

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* In ſome of the former editions, there was a great error ſuffered to paſs here.

§ 9. *Of a* CIRCLE.

A CIRCLE is a plain figure, contained under one line, which is called a circumference, unto which all lines, drawn from a point in the middle or the figure, called the centre, and falling upon the circumference thereof, are all equal the one to the other. The circle contains more ſpace than any plain figure of equal compaſs.

Problem 1. Having the diameter and circumference, to find the area.

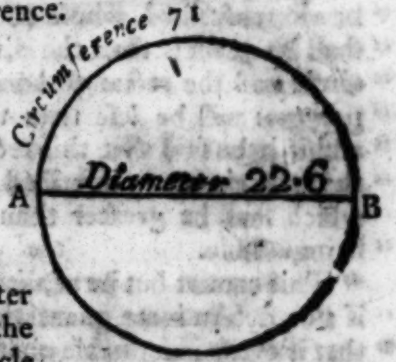
The R U L E.

Every circle is equal to a parallelogram, whoſe length is equal to half the circumference, and the breadth equal to half the diameter; therefore multiply half the circumference by half the diameter, and the product is the area of the circle.

35·5 Half circumference.
11.2 Half diameter.

1065
355
355

401.15 Area



Thus, if the diameter of a circle (that is, the line drawn croſs the circle through the center) be 22.6, and if the circumference be 71, the half of 71 is 35·5, and the half 22.6 is 11.3; which multiplied together, the product is 401. 15, which is the area of the circle.

DEMONSTRATION.

Every circle may be conceived to be a polygon of an infinite number of ſides and the ſemidiameter muſt be equal to the perpendicular of ſuch a polygon, and the circumference of the circle equal to the periphery of the polygon; therefore half the circumference multiplied by half the diameter, gives the area, as aforeſaid.

Or, (with *F. Ignat. Gaſton. Paradies*) " Every " circle is equal to a rectangled-triangle, one of whoſe " legs is the radius, and the other a right line equal " to the circumference of the circle; for ſuch a triangle will be greater than any polygon inſcribed, " and leſs than any polygon circumſcribed, (by the " 24th, 25th, 26th, and 27th articles of the fourth " book of his elements of geometry) and therefore " muſt be equal to the circle.

" For (ſays he) ſhould it be greater than the circle, " be the exceſs as little as it will, a polygon may " be circumſcribed, whoſe difference from the circle, " ſhall be yet leſs than the difference between that " circle and the rectangled-triangle; and that that " polygon will be leſs than the triangle is abſurd; " and if it be ſaid that this rectangled-triangle is leſs " than the circle, an inſcribed polygon may be made " which ſhall be greater than that triangle; which " is impoſſible.

" This cannot but be admitted as a principle, that " if two determinate quantities, A and B are ſuch, " that if every imaginable quantity, which is greater " or leſs than A, is alſo greater or leſs than B, theſe " two quantities A and B muſt be equal.

" And this principle being granted, which is in a " manner ſelf-evident, it may directly be proved, " that the triangle (before mentioned) is equal to the " circle; becauſe every imaginable inſcribed figure, " which is leſs than the circle, is alſo leſs than the " triangle; and every circumſcribed figure greater " than the circle, is alſo greater than the triangle."

Problem 2 Having the diameter of a circle, to find the circumference.

As 7 to 22, ſo is the diameter to the circumference.

Or, as 113 to 355, ſo is the diameter to the circumference.

Or, as 1 to 3.141593, ſo is the diameter to the circumference.

Let the diameter (as in the former circle) be 22.6, this multiplied by 22, and the product is 497.2; which divided by 7, gives 71.028 for the circumference, or (by the ſecond proportion), if 22.6 be multiplied by 355, the product will be 8023; this divided by 11.3, the quotient is 71, the circumference. Or (by the third proportion) if 22.6 be multiplied into 3.141593, the product is 71.0000018, the circumference; which two laſt proportions are the moſt exact.

```
     22.6
       22
   ------
     452
    452
   ------
7) 497.2(71.028

   3.141593
       22.6
  ---------
   18849558
   6283186
  6283186
  ---------
 71.0000018
```

```
         355
        22.6
      ------
        2130
        710
       710
      ------
113) 8028.01(71
     791
     ----
      113
      113
     ----
      ...
```

By Scale *and* Compaſſes.

Extend the Compaſſes from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that extent will reach from 22.6 to 71.

The proportion of the diameter of a circle, to the circumference, was never yet exactly found, notwithſtanding many eminent learned men have laboured very far therein; amongſt which the excellent *Van Culen* hath hitherto outdone all, in his having calculated the ſaid proportion to 36 places of decimals, which are engraven upon his tomb-ſtone in St. *Peter's* Church in *Leyden*; which numbers are theſe,

Diameter.

1.00000.00000.00000.00000.00000.00000.00000

Circumference.

3.14159.26535.89793.23846,26433.83279.50288

Of which large number theſe ſix places, 3.14159, anſwering to the diameter 1.00000, may be ſufficient; of the three proportions, as 7 to 22. 113 to 355, and 1 to 3.14159. I ſhall leave my reader to uſe which of them he pleaſes, but ſhall commend the laſt two as moſt exact, though the firſt be moſt in uſe; but in the following work I ſhall ſometimes uſe one of them, and ſometimes another, but for the moſt part that of *Van Culen*, as being moſt exact

Problem 3. Having the circumference of a circle to find the diameter.

As 1 is to .318309, ſo is the circumference to the diameter.

Or, as 355 to 113, ſo is the circumference to the diameter.

Or, as 22 to 7; ſo is the circumference to the diameter.

Let the circumference be 71, (as in the former circle) if .318309 be multiplied by 71 (as by the firſt proportion) the product will be 22.599939 for the diameter. Or, by the ſecond proportion, 113 multiplied by 71, the product is 8023; which divided by 355, the quotient will be 22.6 the diameter. Or, by the third proportion, 71 multiplied by 7, the product is 497; this divided by 22, the quotient is 22.5909, the diameter.

.318309	113	71
71	71	7
318309	113	22)497(22.59
2228163	791	44
22.599939	355)8023(22.6	57
	710	44
	923	130
	710	110
	2130	200
	2130	198
		2

Thus, by both the firſt proportions, the diameter is 22.6; but by the laſt it falls ſomething ſhort.

By Scale *and* Compaſſes.

Extend the compaſſes from 3.14159 to 1, that extent will reach from 71 to 22.6, which is the diameter ſought.

Or you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the ſame will reach from 71 to 22.6 as before.

NOTE, That if the circumference be 1, the diameter will be .318309.

Problem 4. Having the diameter of a circle, to find the area.

All circles are in proportion one to another, as are the ſquares of their diameters, (by *Euclid*, 12.2.) Now the area of a circle, whoſe diameter is 1, will be 785.398, according to *Van Culen*'s proportion, before mentioned; but for practice .7854 will be ſufficient: therefore,

As 1 (the ſquare of the diameter 1) is to .7854, ſo is 510.76 (the ſquare of 22.6, the diameter of the given circle) to 401.51, (the area of the given circle) But,

According to *Metius*'s Proportion;

As 452 : 355 : : 510. 76 : 401.15, the ſame as before.

But, if you uſe *Archimedes*'s Proportion.

As 14 : 11 : : 510.76 : 401.31; which area is greater than by the two former proportions; though in ſmall circles this is near enough the truth.

See the working of all theſe.

22.6 Diameter of the former circle.
22.6

1356
452
452

510.76 The ſquare of the ſaid diameter.

As 1 : 7854 : : 510.76
.7854

204304
255380
408608
357532

401.150904 The area.

By

By Scale *and* Compaſſes.

The extent from 1 to 22.6, being twice turned over from .7854, will fall at the laſt upon 401.15 the area.

213
4
452

As 452 : 355 : : 510.76
355
255380
255380
153228
452) 181319.80(401.15
1808
519
452
678
452
2260
2260
. . .

As 14 : 11 : : 510.76
11
14) 561836 (401.31
56
18
14
43
42
16
14
2

Problem 5. Having the circumference of a circle, to find the area.

Becauſe the diameters of circles are proportional to the circumference; that is, as the diameter of one circle is to its circumference, ſo is the diameter of another circle to its circumference. Therefore the areas of circles are to one another as the ſquares of the circumferences. And if the circumference of a circle be 1, the area of that circle will be .07958; then the ſquare of 1 is 1, and the ſquare of 71 (the circumference of the former circle) is 5041. Therefore it will be,

Sq. Cir. Area Sq. Circumf.
As 1 : .07958 : : 5041
5041

7958
31832
397900

401.16278 Area

22
4

88

OR THUS:

As 88 : 7 : : 5041 Area.
7

88|35287|400.98 Area.
352

870
792

780
704

76

355
4

1420

Area.
Or, As 1420 : 113 : : 5041 : 401.35.

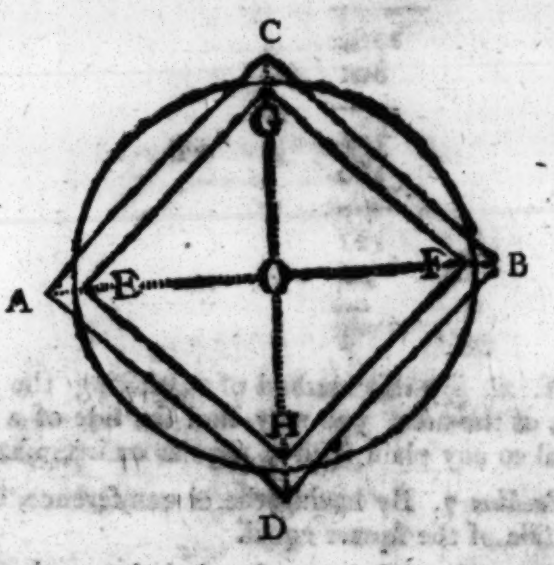
Problem 6. By having the diameter, to find the ſide of a ſquare that is equal in area to that circle.

If the diameter of a circle be 1, the ſide of a ſquare equal thereunto will be .8862 : therefore,

As 1 : : .8862 : : 22.6 (the diameter.)
22.6

53172
17724
17724

To 20.02812 The ſide of the ſquare AC



Let the diameter of the circle EF or GH be 22.6 (as before) to find the ſide of the ſquare C, AD, *&c.* If .8862 be multiplied by 22.6, the product is 20.02812, which is the ſide of a ſquare, equal in area to the circle given; for if 20.02812 be multiplied ſquare wiſe, that is, by itſelf, it will produce 401.1255907344, which is nearly equal to the area found in the laſt *Problem*.

You may find the ſide of the ſquare equal, by extracting the ſquare root out of the area of the given circle.

. . .
401.15 (20.0287295 Side of the ſquare
4

4002) 01.1500
8004

40048) 349600
.... 349600

29216
28034

1182
801

381
360

21
20

1

N. B. By this method of extracting the ſquare root of the area, you may find the ſide of a ſquare equal to any plain figure, regular or irregular.

Problem 7. By having the circumference, to find the ſide of the ſquare equal.

If the circumference of a circle be 1, the ſide of the ſquare equal will be 2821. Therefore,

As 1 : .2821 : : 71 (the circumference)
71

2821
19747

200291 The ſide of the ſquare.

Problem 8. Having the diameter to find the ſide of a ſquare which may be inſcribed in that circle.

If the diameter of a circle be 1, the ſide of the ſquare inſcribed will be .7071. Therefore,

As 1 : 7071 : : 22.6

22.6

42426

14142

14142

To 15.98046 the ſide EG inſcribed.

Or, if you ſquare the ſemidiameter, and double that ſquare, the ſquare root of the doubled ſquare will be the ſide of the ſquare inſcribed. For (by *Euclid*, 1. 47) the ſquare of the hypothenuſe EG is equal to the ſum of the other two legs, EO and OG.

11.3 Semidiameter.

11.3

339

113

113

127.69 The ſquare of EO, which double, becauſe EO=GO :

2

255.38 (15.98 Root, which is the ſide of the ſq

1

25)155

125

309)3038

2781

3188)25700

25504

196

Problem 9. Having the circumference, to find the ſide of a ſquare which may be inſcribed.

If the circumference be 1, the ſide of the ſquare inſcribed will be .2251. Therefore,

As 1 : .2251 : : 71
71

2251
15757

159821 The ſide of the ſquare EG.

Becauſe that in each of the four laſt *Problems*, *viz.* the 6th, 7th, 8th and 9th, there is a proportion laid down, it will be eaſy to work them with ſcale and compaſſes, for if you extend the compaſſes from the firſt to the ſecond, that extent will reach from the third to the fourth. As in the laſt *problem* where the proportion is as 1 to 2251, ſo is 71 to the ſide of the ſquare 15.9821. Here extend the compaſſes from 1 to 2251; that extent will reach from 71 to 15.98; and ſo of the reſt. But the fifth muſt be wrought like the fourth, thus; extend the compaſſes from 1 to 71, that extent turned over the ſame way from ,07958, will fall at laſt upon 401.15

Problem 10. Having the area, to find the diameter.

If the area of a circle be 1, the ſquare of the diameter thereof is 1.2732. Therefore,

```
Area. Sq. Diam. Area. . . . . .
As: 1.2732::401.15   510.744180(25.599 the diam.
        401.15        4
     ----------      ----
        63660      42( 110
       12732            84
      12732           ------
     50928        445( 2674
  ---------------        2225
   510.744180           ------
                  4509(44941
                        40581
                        --------
                 45189( 436080
                        406701
                        --------
                        .29379
```

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 1.2732; that extent will reach from 401.15 to 510.74, *&c.* Then divide the ſpace between 1 and 510.74 into 2 equal parts, and you'll find the middle point at 22.6. Or you may divide the ſpace upon the line of numbers, between 401.55 and .7854, into 2 equal parts, and one of thoſe parts will reach from 1 to 22.6, the diameter ſought.

Problem 11. Having the area, to find the circumference.

If the area of a circle be 1, the ſquare of the circumference will be 12.56637. Therefore,

```
Ar. Sq. Circum. Area. . . . . . . . Circumf.
As1::12.56637::401.15 5040.99932550(70.9999rt.
          408.15          49
       ------------      --------
         6283185    1409) 14099
        1256637           12681
       1256637           ---------
      50265480     14189)141893
   ----------------        127701
   5040.99932550          ---------
                  141989)1419225
                          1277901
                          ----------
                 1419989) 14132450
                          12770901
                          ----------
                          1352540
```

By Scale *and* Compaſſes.

Divide the ſpace between 401.15 and 0.7958, upon the line, into two equal parts; one of thoſe parts will reach from 1 to 71, the circumference ſought.

Problem 12. Having the area, to find the ſide of a ſquare inſcribed.

If the area of a circle be 1, the area of a ſquare inſcribed within that circle will be .6366. Therefore,

As 1 : 401.15 : : .6366

.6366

240690

240690

120345

240690

. . . . . .

255.372090(15.98 root, which is the ſide of the [ſquare ſought.

1

25)155

125

309)3037

2781

188)25620

25504

11690

The ſame reaſon may be given for the laſt proportion that was given before for the proportion of circles to the ſquares of their diameters and circumferences; for not only the ſquares of the diameters and circumferences are in proportion to the circles they belong to, but alſo all figures inſcribed or circumſcribed have the ſquares of their like ſides proportional to the circles they are inſcribed in, or circumſcribed about, and alſo to the figures themſelves: The ſquare of any ſide of one figure is to the area of that figure, as the ſquare of the like ſide of another ſimilar figure is to the area thereof, as you may find proved at large in *Euclid*, *Sturmius's Matheſis Enucleata*, and other authors, but will be too large to inſert in this place.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 401.15, that extent will reach from .6366 to 255.371; the half ſpace between that and 1 is at 15.98, the ſide of the ſquare.

Problem 13. Having the ſide of a ſquare, to find the diameter of the circumſcribing circle.

If the ſide of a ſquare be 1, the diameter of a circle that will circumſcribe that ſquare will be 1.4142. Therefore,

As 1 : 1.4142 : : 15.98
15.98

113136
127278
70710
14142

22.598916 The diameter ſought.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 1.4142, and that extend will reach from 15.98 to 22.6, the diameter ſought.

Problem 14. Having the ſide of a ſquare, to find the diameter of a circle equal.

If the ſide of a ſquare be 1, the diameter of a circle equal thereunto will be 1.128. Therefore,

Side Diam. Side of a ſquare.
As 1 : 1.128 : : 20.0291
1.128

1602328
400582
200291
200291

22.5928248 Diameter.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 1.128; that extent will reach from 20.0291 (the ſide of the ſquare given) to 22.6, the diameter of the circle ſought.

Problem 15. Having the ſide of a ſquare, to find the circumference of the circumſcribing circle.

If the ſide of a ſquare be 1, the circumference of a circle that will encompaſs that ſquare will be 4.443. Therefore,

```
Side Sq. Circum. Side Sq.
As 1 : 4.443 : : 15.98
                 15.98
        ―――――――――――
            35544
           39987
          22215
          4443
        ―――――――――――
          70.99914 the circum.
```

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 4.443, that extent will reach from 15.98 to 71, the circumference.

Problem 16. Having the ſide of a ſquare, to find the circumference of a circle that will be equal thereunto.

If the ſide of the ſquare be 1, the circumference of a circle that will be equal thereunto ſhall be 3.545, Then,

```
As 1 : 3.545 : : 20.0291
                   3.545
              ―――――――――――
                1001455
               801164
              1001455
             600873
              ―――――――――――
             71.0031595 The circum.
```

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 3.545, that extent will reach from 20.0291 to 71, the circumference ſought.

In ſeveral of the foregoing *problems*, where the diameter and circumference is required, the anſwers are not exactly the ſame as the diameter and circumference of the given circle, but are ſometimes too much and ſometimes too little, as in the two laſt problems, where the anſwers in each ſhould be 71, the one being too much, and the other too little. The reaſon of this is, ths ſmall defect that happens to be in the decimal fractions, they being ſometimes too great and ſometimes too little; yet the defect is ſo ſmall, that it is needleſs to calculate them to more exactneſs.

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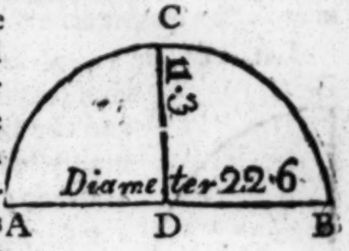
§ X. *Of a* SEMICIRCLE.

TO find the area of a ſemicircle, this is

The RULE.

Multiply the fourth part of the circumference of the whole circle (that is, half the arch line) by the ſemidiameter, the product is the area.



Let ABC be a ſemicircle whoſe diameter is 22.6, and the half circumference, or arch-line ACB, is 35.5, the half thereof is 17.75, which multiply by the ſemidiameter 11.3, and the product is 200.575, the area of the ſemicircle.

17.75 The half arch-line..
11.3 The ſemidiameter..

5325
1775
1775

200.575 The area of the ſemicircle.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 11.3; that extent will reach from 17.75 to 200.575, the area.

If only the diameter of the ſemicircle be given, you may ſay by the *rule of three.*

As 1 is to .3927, ſo is the ſquare of the diameter to the area.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to the diameter 22.6; that extent turned twice from .3927, will reach at the laſt to 200.575.

---

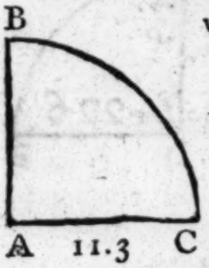
§ XI. *Of a* QUADRANT.

TO find the area of a quadrant, or fourth part of a circle, this is

The RULE.

Multiply half the arch-line of the quadrant, (that is, the eighth part of the circumference of the whole circle) by the ſemidiameter, and the product is the area of the quadrant.

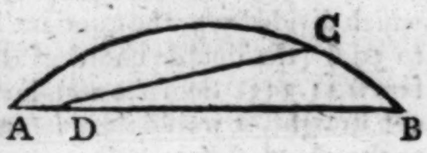
Let ABC be a quadrant, or fourth part of a circle, whoſe radius, or ſemidiameter is 11.3, and the half arch-line 8.875; theſe multiplied together, the product is 100.2875 for the area.



Theſe are the rules and ways commonly given for finding the area of a ſemicircle and quadrant; but, I think, it is a good way to find the area of the whole circle, and then take half that area for the ſemicircle, and fourth part for the quadrant.

Before I proceed to ſhew how to find the area of the ſector, and ſegment of a circle, I ſhall ſhew how to find the length of the arch-line both *geometrically* and *arithmetically*.

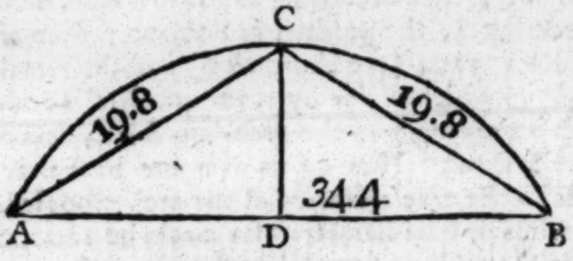
To find the length of the arch-line *geometrically.*



Divide the chord line AB into four equal parts, and ſet one of theſe parts from B to C, and draw a line from C to three of thoſe parts at D; ſo ſhall CD be equal to half the arch-line ACB.

To find the length of the arch-line, *arithmetically.*

Multiply the chord of half the ſegment AC or CB by 8, and from the product ſubtract the chord of the whole ſegment AB, and divide the remainder by 3, the quotient is the arch-line ACB ſought.



19.8 AC
8

158.4
34.4 AB

3) 124

Arch-line 41.333

Another Way.

From the double chord of half the ſegment's arch, ſubtract the chord of the ſegment, one third part of the difference added to the double chord of half the ſegment's arch, the ſum is the arch-line of the whole ſegment.

Thus, if AC 19.8 be doubled, it makes 39.6, from which if you ſubtract 34.4. the remainder is 5.2, which divide by 3, the quotient is 1.733; this added to 39.6 (the double chord of the half ſegment) the ſum is 41.333. So if the arch-line ACB was ſtretched out ſtraight, it would then contain 41.333 ſuch parts as the chord AB contains 34 4 of the like parts.

Theſe two rules may very eaſily be proved out of the table of natural ſines: thus,

Suppoſe (in the former figure) the arch ACB to contain 120 degrees, the natural ſine of half, *viz.* of 60 degrees is 86602, which being doubled is 173204, which is the chord of the whole 120 deg. that is AB. Then, to find the chord of the half arch, *viz.* AC 60 degrees, the half of it 30 deg. the natural ſine thereof is 50000, which doubled make 100000; for the chord AC; then according to the firſt rule, multiply 100000 by 8, the product is 800000; from which ſubtract 173204 (the chord AB) and the remainder is 626796 which divide by 3, the quotient is 208932, which is the length of the arch-line ABC, according to the firſt ral. Now let us examine how near this comes to the true quantity of the arch propoſed: If the radius or ſemidiameter of a circle be 100000 (as it is in the table of ſines) then the circumference will be 628318; and becauſe 120 degrees is the third part of the circle, take the third part of 628318; and becauſe 120 degrees is the third part of the circle, take the third part of 628318, which is 2094.39 the true quantity of the arch ACB in ſuch parts as the ſemidiameter contains 100000, and differs from that before found 507, which is a thing inconſiderable in *Practical Menſuration.* The latter of the foregoing rules gives the quantity of the arch-line too little; and the greater the arch the greater the error. If you know the degrees that are contained in the ſegment's arch, and would have the arch-line very exactly, you may reaſon thus by the *rule of three*;

As the circle's periphery in degrees: is to its periphery

riphery in equal parts : : ſo is the arch in degrees and decimal parts : to the ſame arch in equal parts.

Suppoſe the circumference of a circle be 71, and ſuppoſe the arch to contain 52 degrees, 15 minutes, [the decimal of 15 minutes is .25] then ſay,

Deg. Parts. Deg.
As 360 : 71 : : 52.25
71

5225
36575

36|0] 370|9.75 [10.305 *fere.*
36

109
108

175

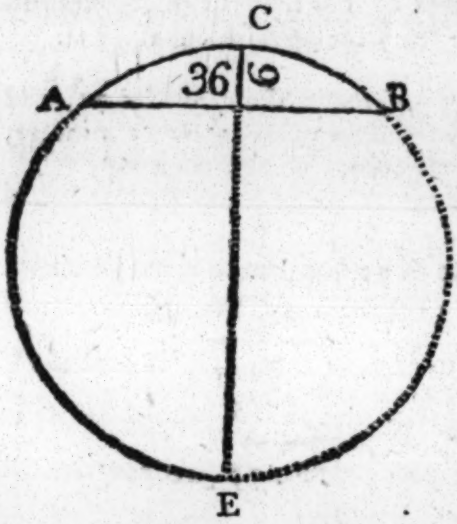
So the 52 Degrees, 15 minutes, will contain 10.305 of ſuch parts as the circumference contains 71.

Thus have I ſhewn ſeveral ways of finding the meaſure of the curve-line of any part of a circle very near the truth. The next thing I ſhall ſhew is,

How to find the diameter of a circle, *by having the chord and verſed ſine of the ſegment, arithmetically.*

Becauſe the chord AB cuts the diameter EC at right angles, therefore the ſemichord AD or DB is a mean proportional line between the parts of the diameter CD and (DE, by *Euclid* 6. 13.) Therefore, if you ſquare the ſemichord AD or DB, and divide that ſquare by the verſed ſine CD, the quotient will be the part of the diameter wanting; to which add the given verſed ſine CD, and the ſum is the diameter ſought.

Exampl



Example.

Let ACB be a ſegment given, whoſe chord AB is 36 and the verſed ſine CD 6; half 36 is 18, which ſquared, makes 324; this divided by 6, the quotient is 54; to which add 6, the ſum is 60, the diameter of the circle CE.

	18	Half the chord.
	18	
	144	
	18	
6)	324	The ſquare of AD.
	54	The part wanting DE.
	6	The verſed ſine CD add.
	60	The diameter CE.

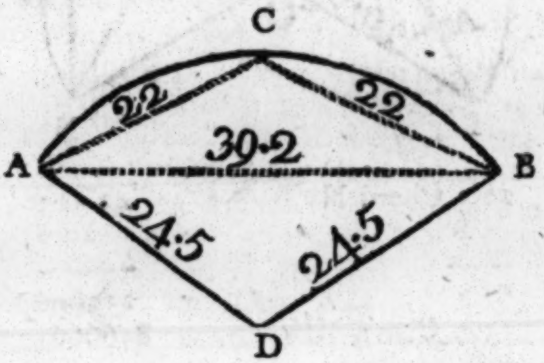
§ XII. *Of a Sector of a* CIRCLE.

A Sector of a circle is comprehended under two radii, or ſemidiameters, which are ſuppoſed not to make one right line, and a part of the circumference; whence a ſector may be either leſs or greater than a ſemicircle. To find the area or ſuperficial content thereof, this is

The R U L E.

Multiply half the arch-line by the ſemidiameter, and the product is the area.

Let ADBC be the ſector of a circle given, whoſe ſemidiameter AD or BD is 24.5, and the arch-line ACB (by the firſt rule, Page 115.) I find to be 45.6 the half thereof, 22.8, being multiplied by 24.5 (the ſemidiameter) the Product is 558.6, which is the area of the ſector ACBD.



22
8

176
39.2 Subtrahend.

3) 136.8

45.6 Arch-line.

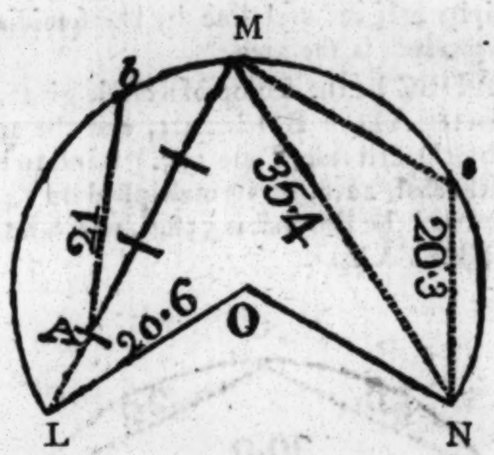
22.8 Half Arch-line
24.5 Semidiameter.

1140
912
456

558.60 The area.

Again. Let LMNO be a ſector greater than a ſemicircle whoſe ſemidiameter LO or NO is 20.6 and line *b a* equal to a fourth part of the arch-line L *b* MN 21, the double whereof is 42, equal to the arch line L*b*M or M*c*N, or by the arithmetical rule in *page* .115, the ſaid arch is found to be 42.333, which multiplied by 20.6, the ſemidiameter makes 872.0598 for the area of the ſector LMNO.

See the following WORK.



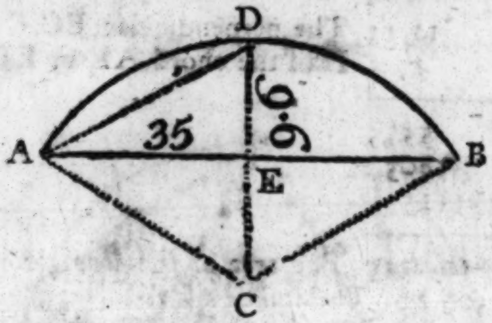
20.3	Chord N*c*	42.333	
8		20.6	
162.4		253998	
35.4	Chord MN	846666	
3) 127.0		872.0598	Area.
42.333	Arch-line.		

§ XIII. *Of the Segment of a* CIRCLE.

A Segment of a circle is a part terminated by a right line leſs than the diameter, called a chord and by a part of the circumference.

To find the area of the ſegment of a circle, you muſt firſt find the center of the whole circle, and draw the two ſemidiameters, thereby completing the ſector, as in the following figure. Then (by the laſt ſection) find the area of the whole ſector CADBC, and then (by ſect. 5.) find the area of the triangle ABC, and ſubtract the area of the triangle out of the area of the ſector, the remainder is the area of the ſegment.

Otherwiſe



Otherwiſe you may, without deſcribing the figure, find the ſemidiameter of the circle by the arithmetical rule, *page* 118, and by the arithmetical rule, *page* 115, find the arch-line; then multiply half the arch-line by the ſemidiameter, ſo have you the area of the ſector. Then ſubtract the verſed ſide from the ſemidiameter, the remainder is the perpendicular of the triangle; and multiply the half chord by the perpendicular, the product is the area of the triangle. Then ſubtract the area of the triangle from the area of the ſector, and the remainder is the area of the ſegment. See the Work.

2) 35=AB.

17.5
17.5

855
1225
175

96) 306.25 (31 9
9.6 add.
182
865 41.5 The diameter of the circle.

1 20.75 The ſemidiameter.
9.6 DE. Subtrahend.

11.15 Remains the perpendic. EC.

11.15 The perpendicular EC.
17.5 Half the chord AE or EB.

5575
7805
1115

195.125 The area of the triangle.
306.25 The ſquare of AE.
92.16 The ſquare of DE, the verſed ſine.

398.41 Sum.

The ſquare root thereof is 19.96 the chord AD.
8

159.68
Subt. 35 the chord AB.

3) 124.68

2) 41.56 the arch-line.

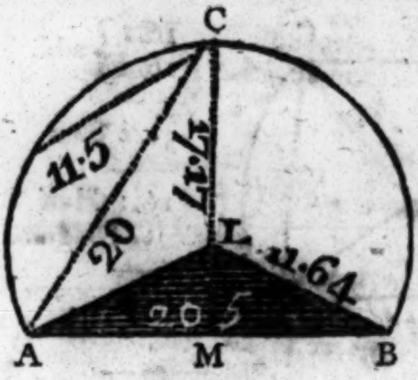
20.78 Half.
20.75 Semidiameter.

10390
14546
4156

From 431.1850 Area of the ſector.
Subtract 195.125 Area of the tri.

Remains 236.060 Area of the ſeg.

Again, Let MACBM be a ſegment greater than a ſemicircle; obſerve the fomer rules in all reſpects, as in the laſt *Example*, only inſtead of ſubtracting the area of the triangle out of the area of the ſector, here you muſt add it thereunto as may plainly appear by the following figure.



11.5
8

92.0
20

3)72

24 Half arch-line.

11.64 Semidiam.
24

4656
2328

17.17
11.64

5.53 LM

Area of the ſector LACBL 279.36
10.25 Half the baſe MA.
5.33 The perpendicular LM.

3075
5125
5125

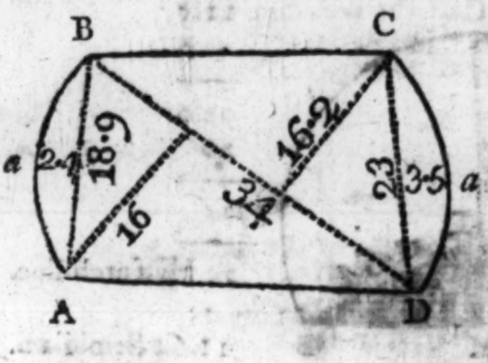
56.6825 The area of the triangle ALM.
279.36 The area of the ſector add.

336.0425 The area of the ſegment ſought.

§. XIV. *Of* COMPOUND FIGURES.

MIXED or compound figures, are ſuch as are compoſed of rectilineal and curvilineal figures together.

To find the area of ſuch mixed figures you muſt find the area of the ſeveral figures, of which the whole compound figure is compoſed, and add all the areas together, and the ſum will be the area of the whole compound figure.



16.2 } add.
16.

32.2 Sum

16.1 Half
34 Diagonal

644
483

547.4 area of the trapezium

10.236 Half the arch-line A*a*B.
14.83 Semidiameter of the arch A*a*B.

30708
81888
40944
10236

151.79988 Area of the ſector.

From 14.83 Semidiameter.
Subtract 3.4 Verſed ſine.

Rem. 11.43 Perpend. of the triangle.
9.45 Half the chord AB.

5715
4572
10287

108.0135 The area of the triangle ſubtracted.
151.7999 From the area of the ſector.

43.7864 The area of the ſegment A*a*BA.

12.19 Half the arch-line C*c*D.
20.64 Semidiameter.

4876
7314
24380

251.6016 The area of the ſector.

From 20.64 The ſemidiameter.
Subtract 3.5 Verſed ſine.

Remaind. 17.14 Perpendic. of the triangle.
11.5 Half the chord DC.

8570
1714
1714

Subtr. 197.110 Area of the triangle.
From 251.602 The area of the ſector.

Rem. 54.492 The area of the ſegment C*c*DC
43.786 The area of the ſegment A*a*BA.
547.4 The area of the trapezium.

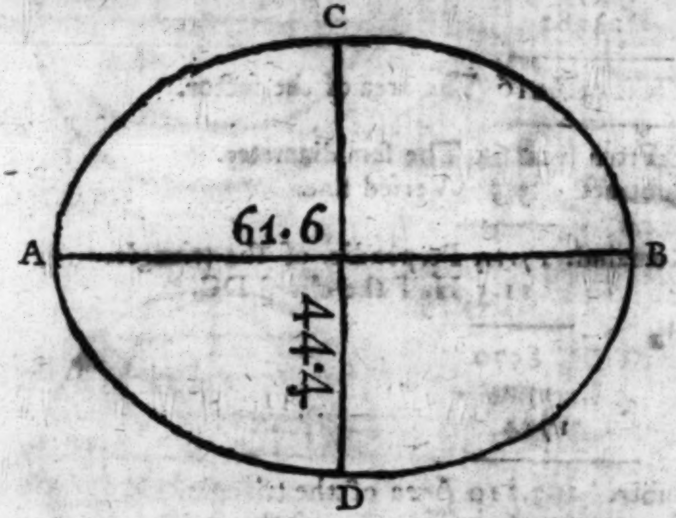
Sum 645.678 The area of the whole.

§. XV. *Of an* ELLIPSIS.

AN ELLIPSIS, or OVAL, is a figure bounded by a regular curve line, returning into itſelf; but of its two diameters, cutting each other in the center, one is longer than the other, in which it differs from the circle. To find the area thereof, this is

The RULE.

Multiply the tranſverſe diameter by the conjugate, and multiply that product by .7854, this laſt product is the area of the oval.



61.6 Tranſverſe diam.
44.4 Conjugate diam.

2464
2464
2464

2735.04 The rectangle.
.7854 Area of unity.

1094016
1367520
2188032
1914528

2148.100416 Area of the oval.

Demonſtration.

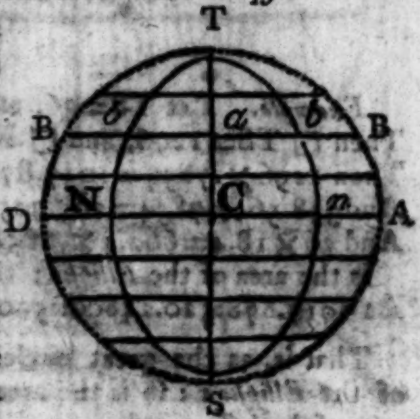
If you circumſcribe any *Ellipſis* with a circle, and ſuppoſe an infinite number of chord lines drawn therein, all parallel to the conjugate diameter, as thoſe in the following figure, then it will be.

As DA the diameter of the circle is to N*n* the conjugate diameter of the *Ellipſis* : : ſo is B*a*B any chord in the circle, : to *bab*, its reſpective ordinate in the *Ellipſis*.

For according to the property of the circle,

it is	1	$aS\times Ta=\square Ba$.
And		by the property of the Ellipſis,
it is	2	$\square TC : \square NC :: aS\times Ta : \square ba$
1, 2,	3	$\square TC : \square NC :: \square Ba : \square ba$.
3, hence	4	$TC : NC :: Ba : ba$.
Conſeq.	5	$TS : 2NC :: 2\,Ba : 2\,ba$.
That is	6	$DA : Nn :: BaB : bab$.

But the ſum of the infinite ſeries of ſuch chords as B*ab* do conſtitute the area of the circle. And the ſum of the like ſeries of their reſpective ordinates, as *bab*, do conſtitute the area of the *Ellipſis*.



Therefore, As TS : to N*n* :: circle's area : to the *Ellipſis* area. But TS : N*n* :: □TS : TS × N*n*; whence it follows, that,

As □ TS : circle's area :: ſo it is TS × N*n*: *Ellipſis* area.

Conſequently, As 1 : to .7854 : : ſo is the rectangle, or product of the tranſverſe and conjugate diameter of any *Ellipſis* to its area.

Hence it is eaſy to conceive, that the ſquare root of the product of the tranſverſe and conjugate diameters, will be the diameter of a circle equal to the *Ellipſis*.

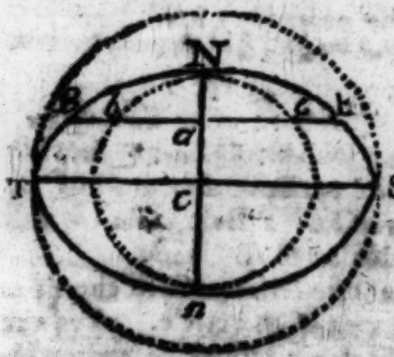
Hence alſo the ſegments of an *Ellipſis*, and its circumſcribing circle (whoſe baſes are parallel to the conjugate diameter, and of the ſame hight) are in proportion one to another, as their baſes are; that is,

As B*a*B : *bab* : : area ſegment BTB : area ſegment *b*T*b*.

Or, As TS : N*n* : : area ſegment BTB : area ſegment *b*T*b*.

The area of every *Ellipſis* is a mean proportion between the areas of its circumſcribing and inſcribed circles.

The



The truth of this may be eaſily deduced from the laſt; for it is already proved, that □ TS : TS × N*n* :: circumſcribing circle's area : *Ellipſis* area.

But □ TS : TS × N*n* : : TS × N*n* : □ N*n* Therefore *Ellipſis* area : inſcribed circle's area : : TS × N*n* : □ N*n*.

Example. Let TS=36, and N*n*=18.4.
Then □ TS=1296, and □ N*n*=338.56.
Then 1296 × .7854=1017.8784 great circle's area.
And 338.56 × .7854=265.905, &c. leſſer circle's area
And 36 × 18.4=662.4 × .7854=520.24896, which is the area of the *Ellipſis*; then it will be,
As 1017.878:520.24896::520.24896:265.905024.

That is, as the great circle's area : is to the area of the *Ellipſis* : : ſo is the area of the *Ellipſis* to the area of the leſſer circle.

From hence it follows, that all ſegments of an *Ellipſis* and its inſcribed circle (whoſe baſes are parallel to the tranſverſe diameter, and have the ſame height) are in proportion one to another, as the area of the *Ellipſis* and circle are.

That is, As the area of the circle : is to the area of the *Ellipſis* : : ſo is the ſegment *b*N*b* : to the ſegment BNB.

Or, N*n* : TS : : area ſegment *b*N*b* : area ſegment BNB.

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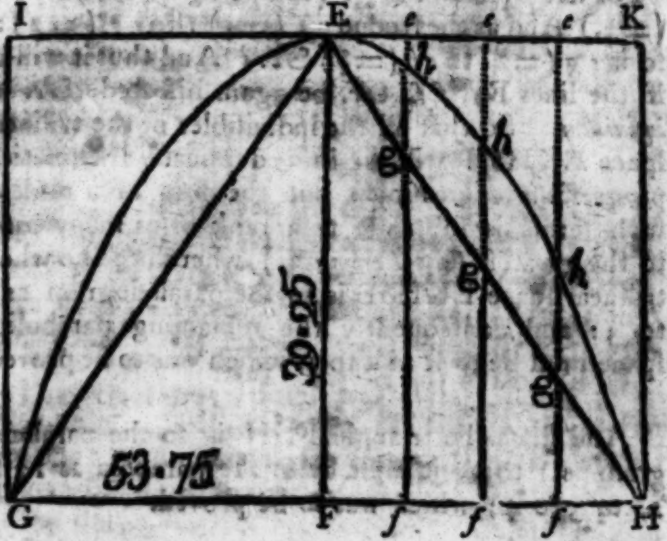
§ XVI. *Of a* PARABOLA.

A PARABOLA is a curvilinear figure, made by the ſection of a cone, being cut by a plane parallel to one of its ſides.

Every parabola is two thirds of its circumſcribing parallelogram; therefore to find the area thereof, this is

The RULE.

Multiply the baſe, or greateſt ordinate, by the perpendicular height, and multiply that product by 2, and divide the laſt product by 3, the quotient will be the area of the parabola.



53.75 the ordinate GH.
39.25 the perpend. EF.

26875
10750
48375
16125

2109.6875
2

3)4219.3750

1406.4583 the area.

Demonſtration. Let FH, the ſemiordinate, be divided into ſour equal parts, or into 8. 16 &c. and thro' the diviſions draw lines, as *e f*, *e f*, &c. parallel to the axis EF. Suppoſe alſo EF to be 4.

Then I ſay, the parabolic ſpace E*b*HF is to the paralellogram EKIH as 2 to 3: but to the triangle EFH as 4 to 3.

For, firſt, gf, gf, gf, *&c.* are in continual arithmetical proportion from the nature of plain triangles.

Secondly, $fe : ge :: ge : he$; but he in the axis EF$=0$, and in the firſt parallel ef it muſt be equal to $\frac{1}{4}$, in the next ef it muſt be equal to $\frac{4}{4}$, in the third to $\frac{9}{4}$ and ſo on, in a duplicate arithmetical progreſſion.

For as ef $(=4)$: to ge $(=1)$ :: ge $(=1)$: to eh $(=\frac{1}{4},)$ And as the ſecond ef $(=4)$: to eg $(=2)$ :: ſo is eg $(=2)$ to eh $(=\frac{4}{4})$ *&c.* And thus it will be if the lines Ff, ff, *&c.* be again biſected, *&c. ad infinitum.* So that all the indiviſibles of the trilinear ſpace EKHhE will be in a duplicate arithmetical progreſſion encreaſing: but the ſum of a rank of ſuch terms is ſubtriple to a rank of as many equal to the greateſt (by *Lemma* 3,) wherefore the whole trilinear ſpace EKHhE is to the parallelogram as 1 to 3: and conſequently the remaining parabolick ſpace muſt be to it as 2 to 3, which was to be proved.

And ſince the triangle FEH is to the parallelogram as 1 to 2, it muſt be to the parabola as $1\frac{1}{2}$ to 2, or 3 to 4; which was to be proved.

Before I proceed to the menſuration of ſolid bodies, I will lay down ſuch *Lemmas* as will be neceſſary to facilitate the demonſtration of ſuch ſolids.

LEMMA I.

In any ſeries of equal numbers, (repreſenting lines or other quantities) 1, 1, 1, 1, *&c.* or 2, 2, 2, 2, *&c.* or 3, 3, 3, 3, *&c.* if one of the terms be multiplied into the number of terms, the product will be the ſum of all the terms in the ſeries.

LEMMA

LEMMA II.

If a ſeries of numbers, in arithmetical progreſſion, begin with a cipher, and the common difference be 1, as 0, 1, 2, 3, *&c.* (repreſenting a ſeries of lines or roots beginning with a point) if the laſt term be multiplied into the number of terms, the product will be double the ſum of all the ſeries.

That is putting L= the laſt term, N= the number of terms, and S= the ſum of all the ſeries. Then will NL=2 S; conſequently, $\frac{1}{2}$ NL=S *viz.* one half of ſo many times the greateſt term as there are the number of terms in the ſeries.

Thus, $\left\{\begin{array}{l}0+1+2+3+4=\overline{10} \text{ the ſum}=\frac{1}{2}\text{ NL.}\\ \overline{4+4+4+4+4}=20=\text{NL.}\end{array}\right.$

LEMMA III.

If a ſeries of ſquares, whoſe ſides or roots are in arithmetical progreſſion, beginning with a cipher, *&c.* be infinitely continued, the laſt term being multiplied into the number of terms, will be triple to the ſum of all the ſeries, *viz.* NLL = 3S; or $\frac{1}{3}$ NLL=S.

That is, the ſum of ſuch a ſeries will be one third of the laſt, or greateſt term, ſo many times repeated as there are numbers of terms in their ſeries.

Inſtances in ſquare numbers.

1 $\left\{\frac{0+1+4}{4+4+4}\right.=\frac{5}{12}=\frac{1}{3}+\frac{1}{12}.$

2 $\left\{\frac{0+1+4+9}{9+9+9+9}\right.=\frac{14}{36}=\frac{7}{18}=\frac{1}{3}+\frac{1}{18}.$

3 $\left\{\frac{0+1+4+9+16}{16+16+16+16+16}\right.=\frac{30}{80}=\frac{3}{8}=\frac{1}{3}+\frac{1}{24}.$ $\frac{2}{24}$

From theſe inſtances it is evident, that as the number of terms in the ſeries increaſe, the fraction, or exceſs above one third does decreaſe, the ſaid exceſs always being $\frac{1}{6N-6}$, which if we ſuppoſe the ſeries to be infinitely continued, will quite vaniſh, and become nothing at all.

LEMMA IV.

If a ſeries of cubes, whoſe roots are in arithmetical progreſſion, beginning with a cipher, &c. (as above) be infinitely continued, the ſum of all the ſeries will be one fourth NLL=S.

That is, one fourth of the laſt term ſo many times repeated as there are numbers of terms.

Inſtances in Cube numbers.

If 0, 1, 2, 3, 4, 5, &c. be the roots of the cubes.

$$1\ \left\{\frac{0+1+8+27}{27+27+27+27}=\frac{36}{108}=\frac{4}{12}=\frac{1}{4}+\frac{1}{12}.\right.$$

$$2\ \left\{\frac{0+1+8+27+64}{64+64+64+64+64}=\frac{100}{320}=\frac{10}{32}=\frac{5}{16}=\frac{1}{4}=\frac{1}{16}\right.$$

$$3\ \left\{\frac{0+1+8+27+64+125}{125+125+125+125+125+125}+\frac{225}{750}=\frac{45}{150}\right.$$

$$(=\frac{6}{20}=\frac{1}{4}+\frac{1}{20}.$$

From theſe *Examples* it plainly appears, that as the number of terms in the ſeries increaſes, the fraction or exceſs above one fourth decreaſes, the exceſs being always $\frac{1}{4N-4}$, which, if we ſuppoſe the ſeries to be infinitely continued, will become infinitely ſmall, or nothing.

LEMMA

LEMMA V.

If a ſeries of biquadrats, whoſe roots are in arithmetical progreſſion, beginning with a cipher, *&c.* as before, be infinitely continued, the ſum of all the terms in ſuch a ſeries will be one fifth NLLLL.

The truth of this may be manifeſt by the like proceſs, as in the foregoing *lemmas*, and ſo on for higher powers.

LEMMA VI.

The ſum of an infinite progreſſion, whoſe greateſt term is a ſquare number, the others decreaſing by odd numbers, *viz.* 1, 3, 5, *&c.* is in ſubſeſquialtern proportion of the ſum of the like number of equal terms, that is, as 2 to 3.

Inſtances in ſuch progreſſions.

1 $\left\{\frac{9+8+5}{9+9+9}\right.=\frac{22}{27}=\frac{2}{3}+\frac{4}{9}$

2 $\left\{\frac{16+15+12+7}{16+16+16+16}\right.=\frac{50}{64}=\frac{2}{3}+\frac{11}{32}$

3 $\left\{\frac{25+24+21+16+9}{25+25+25+25+25}\right.=\frac{95}{125}=\frac{2}{3}+\frac{7}{27}$

4 $\left\{\frac{36+35+32+27+20+11}{36+36+36+36+36+36}\right.=\frac{161}{216}=\frac{2}{3}+\frac{17}{72}$

From theſe *examples* it plainly appears, that as the number of terms in the ſeries increaſes, the fraction or exceſs above $\frac{2}{3}$ decreaſes: and if we ſuppoſe the ſeries to be infinitely continued, that exceſs will quite vaniſh, and the ſum of the infinite ſeries will be $\frac{2}{3}$ of ſo many equal, to the greateſt.

CHAP.

CHAP. II.

Menſuration of SOLIDS.

SOLID bodies are ſuch as do conſiſt of length, breadth, and thickneſs; as ſtone, timber, globes, bullets, &c.

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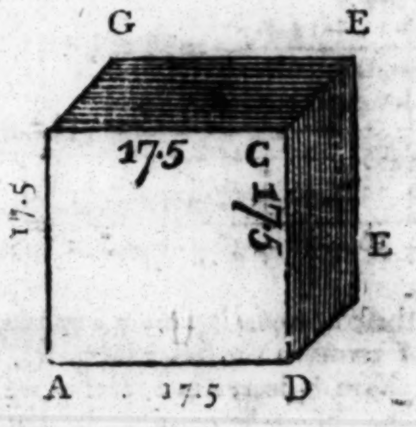
§ I. *Of a* CUBE.

A CUBE is a ſquare ſolid comprehended under ſix geometrical ſquares, being in the form of a dye. To find the ſolid content, this is

The RULE.

Multiply the ſide of the cube into itſelf, and that product again by the ſide, the laſt product will be the ſolidity, or ſolid content of the cube.



17.5
17.5

875
1225
175

306.25
17.5

153125
214375
30625

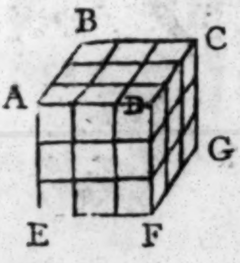
5359.375 Solid content of the cube.

Suppoſe ABCDEFG a cubical piece of ſtone or wood, each ſide thereof being 17 inches and an half: multiply 17.5, by 17.5, and the product is 306.25; which being multiplied by 17.5, the laſt product is 5359.375, which is 5359 ſolid inches, and 375 parts. To reduce the ſolid inches to feet, divide by 1728 (becauſe ſo many cubical inches is a foot) and the ſolid feet in the cube will be 3, and 175 cubical inches remain.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 17.5, that extent turned over twice from 17.5, will reach to 5359, the ſolid content in inches. Then extend the compaſſes from 1728 to 1, that extent turned the ſame way from 5359, will reach to 3.1 feet.

DEMONSTRATION.



If the ſquare ABCD be conceived to be moved down the plain ADEF, always remaining parallel to itſelf, there will be generated, by ſuch a motion, a ſolid having ſix plains, the two oppoſite whereof will be equal and

parallel

parallel to each other; whence it is called a parallelopipedon, or ſquare priſm. And if the plain, ADEF be a ſquare equal to the generating plain ABCD, then will the generated ſolid be a cube. From hence ſuch ſolids may be conceived to be conſtituted of an infinite ſeries of equal ſquares, each equal to the ſquare ABCD; and AE or DF will be the number of terms. Therefore if the area of ABCD be multiplied into the number of terms, AE the product is the ſum of all the ſeries, (*per Lemma* I) and conſequently the ſolidity of the parallelopipedon or cube. Or if the baſe ABCD, being divided into little ſquare areas, be multiplied into the height AE, divided by a like meaſure for length, after this way you may conceive as many little cubes to be generated in the whole ſolid, as is the number of the little areas of the baſe multiplied by the number of diviſions the ſide AE contains. Thus if the ſide of the baſe AB be 3, that multiplied into itſelf is 9, which is the area of the ſquare baſe ABCD; then if AE be likewiſe 3, multiply 9 by 3, and the product is 27; and ſo many little cubes will this ſolid be cut into, if you conceive it to be cut as the lines direct.

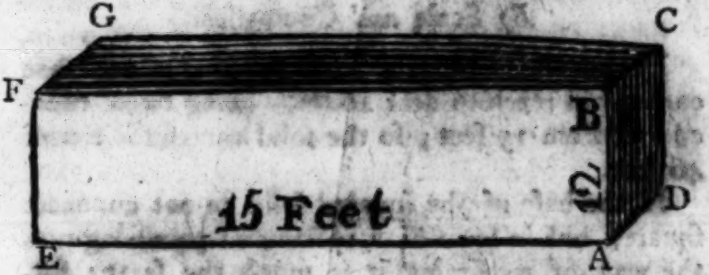
From this demonſtration it is very plain, that if you multiply the area of the baſe of ony parallelopipedon into its length or height, that product will be the ſolid content of ſuch a ſolid.

---

§ II. *Of a* PARALLELOPIPEDON.

LET ABCDEFG be a parallelopipedon, or ſquare priſm, repreſenting a ſquare piece of timber or ſtone, each ſide of its ſquare baſe ABCD being 21 inches, and its length AE 15 feet.

First,



Firſt, then, multiply 21 by 21, the product is 441, the area of the baſe in inches; which multiplied by 180, the length in inches, and the product is 79380, the ſolid content in inches. Divide the laſt product by 1718, and the quotient is 45.9, that is 45 ſolid feet, and 9 tenths of a foot. Or thus, multiply 441 by 15 feet, and the product is 6615; divide this by 144, and the quotient is 45.9, the ſame as before.

Or thus, by multiplying feet and inches.

Multiply 1 foot 9 inches, by 1 foot 9 inches, and the product is 3 feet, 0 inches, 9 parts; this multiplied again by 15 feet, gives 45 feet, 11 inches, 3 parts; that is, 45 feet, and $\frac{11}{12}$ of a foot, and $\frac{1}{4}$ of $\frac{1}{12}$.

See the Work of all theſe,

```
        21               441              F.  I.
        21                15              1   9
      ----             -----              1   9
        21              2205             ---------
       42               441               1   9
      ----             -----              1   3   9
       441       144)6615(45.9           ---------
       180             -----              3   0   9
      ----               885             15
     35280              1350             ---------
     441               -----              4   0   0
     -----                54                  7   6
1728)79380(45.9                               3   9
     6912                                ---------
     -----                               45  11   3
     10260
      8640
     -----
     16200
     15552
     -----
       648
```

By Scale *and* Compaſſes.

Extend the compaſſes from 12 to 21, and that extent will reach to near 46 feet, being twice turned over from 15 feet; ſo the ſolid content is almoſt 46 feet.

If the baſe of the ſquared ſolid be not an exact ſquare, but in form of a rectangled parallelogram, the way of meaſuring it is much the ſame; for, firſt you muſt find the area of the baſe, by multiplying the breadth by the depth, and then multiply that area by the length of the piece, as before; thus,

If a piece of timber be 25 inches broad, 9 inches deep and 25 feet long, how many ſolid feet are contained therein ?

```
      25
       9            F.  I.
    ----            2   1
     225            0   9
      25          ---------
    ----            1   6   9
    1125           25
    450           ---------
    -----          25   0   0
144)5625(39.       12   6   0
    432             1   0   6
    ----            0   6   3
     1305         ---------
     1296          39   0   9
     ----
       09 Anſwer 39 Feet.
```

By Scale *and* Compaſſes.

Firſt find a mean geometrical proportional between the breadth and the debth; which to do upon the line of numbers, you muſt divide the ſpace upon the line between the breadth and depth, into two equal parts; that middle point will be the mean proportional ſought; thus the middle point between 25 and 9 is at 15; ſo is 15 a mean proportional between 9 and 25:

for, as 9 : 15 : : 15 : 25 ; ſo a piece of timber of 15 inches ſquare is equal to a piece 25 inches broad and 9 inches deep. So then, if you extend the compaſſes from 12 to 15, that extent turned twice over from 25 feet, the length, will reach to 39 feet, the content.

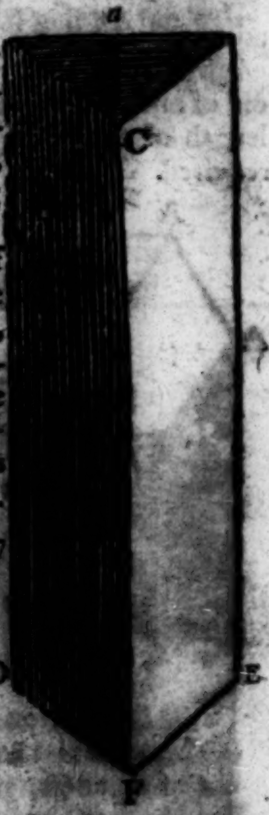
---

§ III. *Of a Triangular* PRISM.

A Priſm is a ſolid contained under ſeveral plains, and having its baſes alike, equal and parallel: The ſolid content of a priſm (whether triangular or multangular) is found by multiplying the area of the baſe into the length or height, and the product is the ſolid content.

Let ABCDEF be a triangular priſm, each ſide of the baſe being 15.6 inches, the perpendicular thereof C*a* is 13.51 inches, and the length of the ſolid 19.5 feet.

Multiply the perpendicular of the triang. 13.51 by half the ſide 7.8, and the product is 105.378, the area of the baſe; which multiply by the length 19.5, and the product is 2054.871, which divide by 144, and the quotient is 14.27 feet, *fere*, the ſolid content.



```
      13.51   144)2054.87(1427
        7.8        144
     ------       ----
     10808         614
     9457          576
    -------       ----
    105.378        388
       19.5        288
   --------       ----
     526890        1007
    948402         1008
   105378
   ---------
   2054.8710
```

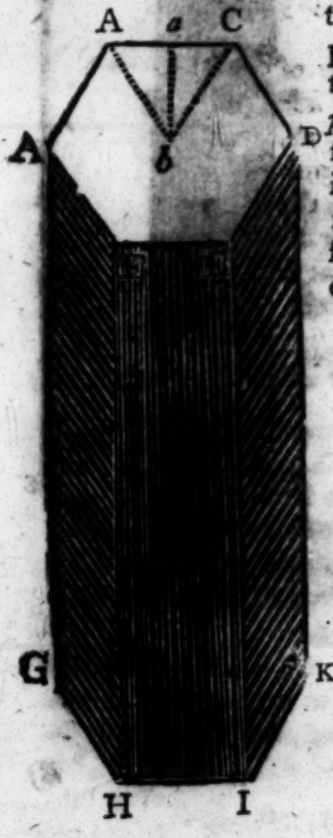
By

By Scale *and* Compaſſes.

Firſt, find a mean proportional between the perpendicular and half ſide, (as before taught) by dividing the ſpace upon the line, between 13.51 and 7.8, into two equal parts; ſo ſhall you find the middle point between them to be at 10.26, which is the mean proportional ſought: by this means the triangular ſolid is brought to a ſquare one, each ſide being 10.26 inches.

Then extend the compaſſes from 12 to 10.26, that extent turned twice downwards from 19.5 feet, the length, will at laſt fall upon 14.27, which is 14 feet and a little above a quarter.

Let ABCDEFGHIK repreſent a priſm whoſe baſe is a hexagon, each ſide thereof being 16 inches, and the perpendicular, from the centre of the baſe to the middle of one of the ſides *(ab)* is 13.84 inches, and the length of the priſm is 15 feet; the ſolid content is required.



Multiply half the ſum of the ſides 48 by 13.84, and the product is 664.32, the area of the hexagonal baſe, (by § 8, *p.* 86) which multiply by 15 feet, the length, the product is 9964.80, which divide by 144, the quotient will be 69.2 feet, the ſolid content required.

```
      13.84
         48
     ------
      11072
      5536
     ------
     664.32  Area of baſe
         15
     ------
     332160
     66432
     ------
144)9964.80(69.2
    864
    ----
    1324
    1296
    ----
     288
     288
    ----
     ...
```

By

By Scale *and* Compaſſes.

Firſt, find a mean proportional between the perpendicular and half the ſum of the ſides, that is divide the ſpace between 13.84 and 48, and the middle point will be 25.77, then extend the compaſſes from 12 to 25.77, that extent will reach (being twice turned over) from 15 feet, the length, to 69.2 feet, the content.

To find the ſuperficial content of any of the forementioned ſolids, you muſt take the girt of the piece and multiply by the length, and to that product add the two areas of the baſes, the ſum will be the whole ſuperficial content. *Example* of the hexagonal priſm laſt mentioned: the ſum of the ſides being 96, and the length 16 feet, that is 180 inches, which multiplied by 96, the product is 17280 ſquare inches, to which add twice 664.32, the areas of the two baſes, and the ſum is 18608.64, the area of the whole, which is 129.22 feet.

```
   180
    96          144) 18608.64 (129.22
  ----               --------
  1080               420
 1620                1328
 -----                326
 17280                 384
   664.32            -----
   664.32               96
 --------
 18608.64
```

The ſuperficial content of the whole ſolid is 129.22 feet.

By Scale *and* Compaſſes.

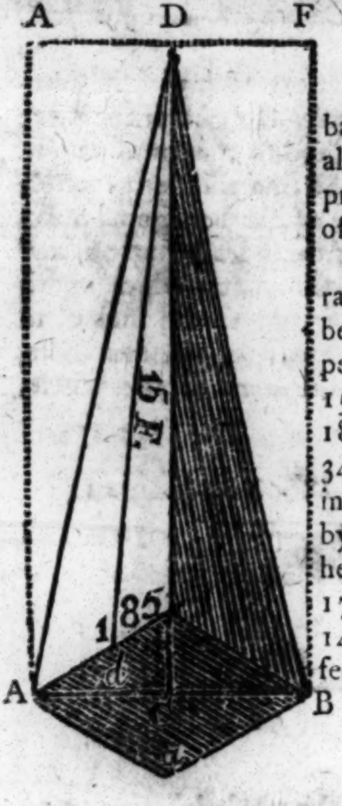
Extend the compaſſes from 144 to 180, that extent will reach from 96 to 120 feet: Then, to find the area of the baſe, extend the compaſſes from 144 to 13.84 that extent will reach from 48 to 4.6 feet; add 120 feet, and twice 4.5 feet, and it make 129.2 feet, the ſuperficial content, as before.

The demonſtration of theſe laſt ſolids will be the ſame as in the firſt ſection; for as in that, ſo in theſe the area of the baſe is multiplied into the length to find the content, and the ſame reaſon is given for one as for the other.

§ IV.

§ IV. *Of a* PYRAMID.

A PYRAMID is a ſolid figure, whoſe baſe is a polygon, and whoſe ſides are plain triangles, their ſeveral tops meeting together in one point. To find the ſolid content thereof, this is



The RULE.

Multiply the area of the baſe by a third part of the altitude, or length, and the product is the ſolid content of the pyramid.

Let ABD be a ſquare pyramid, each ſide of the baſe being 18.5 inches, and the perpendicular height CD is 15 feet: Multiply 18.5 by 18.5, and the product is 342.25, the area of the baſe in inches: which multiplied by 5, a third part of the height, and the product is 1711.25: this divided by 144, the quotient is 11.88 feet, the ſolid content.

		F.	I.	Pts.	
18.5		1	6	6	
18.5		1	6	6	
925		1	6	6	
1480			9	3	
185				9	3
342.25	Area of the baſe.	2	4	6	3
5					5
144)1711.25(1188 Content.		11	10	7	3

By

By Scale *and* Compaſſes.

Extend the compaſſes from 12 to 18.5 inches, that extent turned twice over from 5 feet (a third part of the height) will fall at laſt upon 11.88 feet, the ſolid content.

To find the ſuperficial content.

Multiply the ſlant height (or perpendicular of one of the triangles) by half the periphery of the baſe 37, and the Product is 6668.88, which divided by 144, the quotient is 46.31 feet, the ſuperficial content of all but the baſe; then to that add 2.38 feet, the baſe, and it makes 48.69 feet, the whole ſuperficial content.

```
  180.25 The ſlant height d D.
      37
  ------                            144)342.25(2.38
  126168                                288
  54072                                 ----
  ------                                542
144)6668.88(46.31                       432
   576       2.38                       ----
   ---       ----                       1105
   908      48.69The whole content.     1152
   864                                  ----
   ---
   448
   432
   ---
    168
    144
    ---
     24
```

By Scale and Compaſſes.

Extend the compaſſes from 144 to 180.24, that extent will reach from 37 to 46.31 feet, the area of the four triangles; and extend the compaſſes from 144 to 18.5 (one ſide of the baſe) that extent will reach from 18.5 to 2.38 *fere*; which added to the other, the ſum is 48.69, the whole ſuperficies.

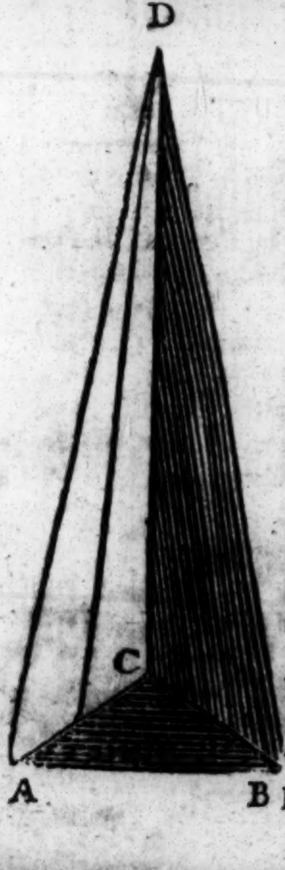
DEMON-

DEMONSTRATION.

Every pyramid is a third part of the priſm, that hath the ſame baſe and height, (by *Euclid*, 12.7)

That is the ſolid content of the pyramid ABD (in the laſt figure) is one third part of its circumſcribing priſm ABEF.

For every pyramid that hath a ſquare baſe (ſuch as A*a* B*b* in the laſt figure) is conſtituted of an infinite ſeries of ſquares, whoſe ſides or roots are continually increaſing in arithmetical progreſſion, beginning at the vertex or point D, its baſe A*a*B*b* being the greateſt term, and its perpendicular height CD is the number of all the terms: But the laſt term multiplied into the number of terms, the product will be triple the ſum of all the ſeries, (by *Lemma* III.) conſequently $\frac{NLL}{3}$ =S. And S is equal to the ſolid content of the pyramid. From hence it will be eaſy to conceive, that every pyramid is $\frac{1}{3}$ of its circumſcribing priſm, (that is, of a priſm of equal baſe and altitude) what form ſoever its baſe is of, *viz.* whether it be ſquare, triangular, pentagular, &c. Yo may very eaſily prove a triangular pyramid to be a third part of a priſm of equal baſe and altitude, by cutting a triangular priſm of cork, and then cut that priſm into three pyramids, by cutting diagonally, as I have ſeveral times done, to ſatisfy myſelf and others.



Let ABCD be a triangular pyramid, each ſide of the baſe being 21.5 inches, and its perpendicular height 16 feet; the content ſolid and ſuperficial is required.

Firſt, find the area of the baſe, by multiplying half the ſide by the perpendicular let fall from the angle of the baſe to the oppoſite ſide; which perpendicular will be found to be 18.62; the half thereof is 9.31, which multiplied by 21.5, the Product is 200.165 Inches, the Area of the Baſe. Then becauſe the Altitude 16 cannot exactly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multiply it by 16, and the Product is 1067.52 which divided by 144, the Quotient is 7.41 Feet, the ſolid Content.

9.31 Half the Perpen.		F.	I.	Pts.	
21.5 The Side.	Side	1	9	6	
	Half Perpen.		9	4	
4655					
931		1	4	1	6
1862				7	2
3) 200.165 Area Baſe.	Area Baſe	1	4	8	8
					4
66.72 A third Part.					
16 Height.		5	6	10	8
					4
40032					
6672	3)	22	3	6	8
144)1067.52 (7.41 Sol. Cont.	Cont.	7	5	2	2
1008					

In caſting this up by Feet and Inches
595 inſtead of multiplying by 16, the
576 Height, I break 16 into two ſuch
Numbers, as being multiplied together
192 the Product may be 16, *viz.* into 4
144 and 4, and firſt multiply by one, and
then the other; a third Part of the
48 laſt Product is the Content.

By Scale *and* Compaſſes.

Firſt, find a geometrical mean Proportional, (as before directed) by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the mean Proportional ſought. Then extend the compaſſes from 12 to 14.15, that Extent (turned twice over from 16 Feet) will fall at laſt upon 22.23; a third Part thereof is 7.41 Feet, the Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe, and to that Product add the Area of the Baſe, the Sum is the whole ſuperficial Content.

192.1 Inches, the ſlant Height *d* D.
Half Periphery 32.25=21.25+10.75

9605
3842
3842
5763

6195.225 Inches, the Area of all but the [Baſe.
200.165 Area of the Baſe add.

144)6395.390)44.41 Feet, the whole Cont.
576

635
576

593
576

179
144

35

By Scale *and* Compaſſes.

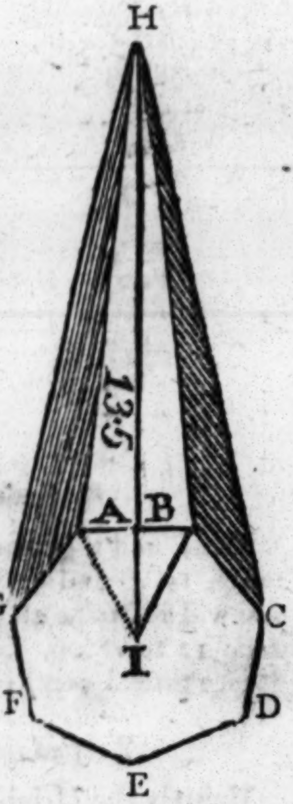
Extend the Compaſſes from 144 to 192.1, that Extent will reach from 32.25 (half the Periphery of the Baſe) to 43.02 Feet, the Content of the upper Part.

And extend the Compaſſes from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Baſe, which added to the other makes 44.41 Feet, the Content of the Whole.

Let ABCDEFGH be a pyramid, whoſe baſe is a heptagon, each ſide thereof being 15 inches, and the perpendicular of the heptagon is 15.58 inches, and the perpendicular height of the piramid HI is 13.5 feet; the content ſolid and ſuperficial is required.

Multiply 15.58 (the perpendicular) by 52.5, half the ſum of the ſides of the heptagon, and the product is 817.95, which multiplied by 4.5 *viz.* $\frac{1}{3}$ of the height, and the product is 3680.775.

Then divide this laſt product by 144, and the quotent is 25.56 feet, the content.



15.58 The Heptagon's Perpendicular.
52.5 The Half Sum of the Sides.

7790
3116
7790

817.950
4.5 A third part of the height.

4089750
3271800

---

144) 3680.7750(25.56 Solid Feet.
288

800
720

807
720

877
864

13

By Scale *and* Compaſſes.

Firſt, find a geometrical mean Proportional between 15.58 and 52.5 (as is before directed) which you will find to be 28.06; then Extend the Compaſſes from 12 to 28.06, that Extent will reach from 4.5 (twice turned over) to 25.56 Feet.

To find the ſuperficial Content.

Multiply the Height taken from the Middle of one of the Sides of the Baſe 162.75 Inches, by the half Sum of the Sides 52.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part.

162.75
52.5

81375
32550
81375

144) 8544.375 (59.335 Feet.
5.68 Baſe add.

1344
483
517
855

135

65.015

144)817.95 (5.68
979
1155

3

By Scale *and* Compaſſes.

Extend the Compaſſes from 144 to 162.75, that Extent will reach from 52.5 to 59.333 Feet.

And extend the Compaſſes from 144 to 15.58, the Perpendicular of the Heptagon, the Extent will reach from 52.5 to 5.68 Feet, the Content of the baſe; which add to the former, the ſum is 65.015, the whole Superficial Content.

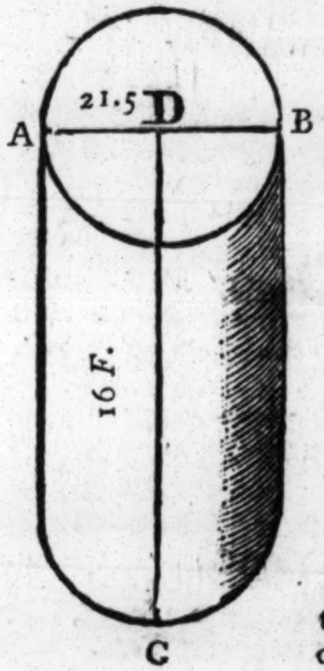
---

§ V. *Of a* CYLINDER.

A CYLINDER is a round Solid, having its Baſes circular, equal and parallel, in Form of a Rolling Stone uſed in Gardens. To find the ſolid Content thereof, this is

The RULE.

Multiply the Area of the Baſe by the Length, and the Product is the ſolid Content.



Let ABC be a Cylinder, whoſe Diameter AB is 21.5, Inches, and the Length CD is 16 Feet, the ſolid Content is required.

Firſt, ſquare the Diameter 21.5 and it makes 462.25; which multiply by .7854 and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8164. Divide this laſt Product by 144, and the Quotient is 40.34 Feet, the ſolid Content.

By Scale *and* Compaſſes.

Extend the Compaſſes from 13.54 to 21.5, the Diameter, that Extent (turned twice over from 16, the Length) will at laſt fall upon 30.34, the ſolid Content.

To find the ſuperficial Content.

Firſt, (by Ch. 1 Sect. IX. Prob. 2.) find the Circumference of the Baſe 67.54, which multiplied by 16, the Product is 1080.64; which divided by 12, the Quotient is 90.05 Feet, the Curve Surface: to which add 5.04 Feet, the Sum of the two Baſes, and the Sum is 95.09 Feet, the whole ſuperficial Content.

67.54
16
———
40524
6754
———
12)1080.64
———
90.05

90.05 } add.
5.04 }
———
95.09

363.05
2
———
144)726.10(5.04
———
610
———
34

By

By Scale *and* Compaſſes.

Extend the compaſſes from 12 to 67.54 (the circumference) that extent will reach from 16, the length, to 90.05 feet, the curve-ſurface.

And extend the compaſſes from 12 to 21.5, (the diameter) that extent (turned twice from .7854) will at laſt fall upon 2.52 feet, the area of one baſe; which doubled is 5.04; this added to the curve-ſurface makes 95.09 feet, the whole ſuperficial content.

DEMONSTRATION.

The ſolid content of every Cylinder is found by multiplying the area of its baſe into its height, as aforeſaid: For every right Cylinder is only a round Priſm, being conſtituted of an infinite ſeries of equal circles; that of its baſe, or end, being one of the terms, and its height CD (in the former figure) is the number of all the terms: Therefore the area of its baſe AB being multiplied into CD, will be its ſolidity (by *Lemma* I.) Let $D=AB$, and $H=CD$.

Then $.7854\, DD \times H =$ is ſolidity.

---

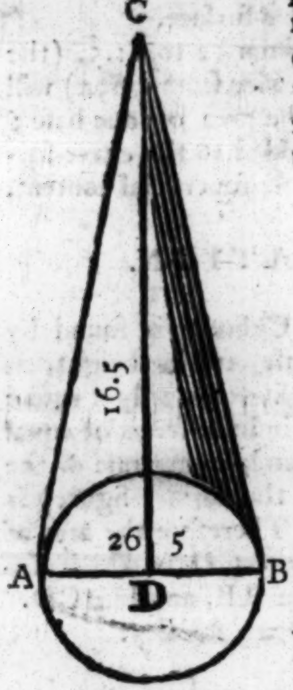
§ VI. *Of a* CONE.

A CONE is a Solid, having a circular baſe, and growing ſmaller and ſmaller till it ends in a point, which is called the vertex, and may be nearly repreſented by a Sugar-loaf. To find the ſolidity thereof, this is

The RULE.

Multiply the area of the baſe by a third part of the perpendicular height, and the product is the ſolid content.

Let ABC be a Cone, the Diameter of whoſe baſe AB is 26.5 inches, and the height of the cone DC is 16.5 feet. Firſt, ſquare the diameter 26.5, and it is .702.25 ; which multiply by .7854, and the product is 551.54715 ; which multiply by 5.5, and the product is 3035.47825 ; which divided by 144, the quotient is 21.07 *fere*, the ſolid content of the cone.



```
         26.5 The diameter.
         26.5
        -----
         1325
        1590
        530
       ------
       702.25 The ſquare.
        .7854
      -------
       280900
      351125
     561800
    491575
   ------------
   551.54|7150 Area of the baſe.
          5.5 Third part of the height
 ------------
    275770
   275770
  -----------
144)3033.47|0(21.06 feet, the content.
  -----------
    153
     947
    -----
     83
```

By Scale *and* Compaſſes.

Extend the compaſſes from 13.54 to 26.5, (the diameter) that extent turned twice over from 5.5 a third part of the height) will at laſt fall upon 21.06 feet the content.

By

To find the ſuperficial Content.

Multiply half the Circumference 41.626 by the ſlant height AC 198.46, the Product is 8261.09596 which divided by 144, the Quotient is 57.37, *fere*, the Curve-Surface; to which add the Baſe, the Sum is 61.2, the ſuperficial Content.

41.626 The half Circumfer. of the Baſe.
198.46 Inches, the ſlant Height.

249756
166504
333008
374634
41626

---

144)8261.09596(57.37 Feet, *fere*.
3.83 The Baſe add.

1061
530
989

61.20 The whole Content.

144)551.54(383

1195
434

2

By Scale *and* Compaſſes.

Extend the compaſſes from 144 to 198.46, that Extent will reach from 41.626 to 57.37 Feet, the Curve-Surface.

And extend the compaſſes from 12 to 26.5 (the Diameter) that Extent, turned twice over from .7854, will at laſt fall upon 3.83 Feet, the Baſe; which added to 57.37, the Sum is 61.2 Feet, the ſuperficial Content.

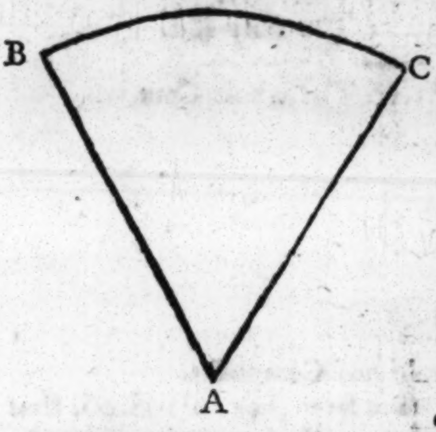
DEMONSTRATION.

Every Cone is the third Part of a Cylinder of equal Baſe and Altitude. The Truth of this may eaſily be conceived by only conſidering, that a Cone is but a round Pyramid, and therefore it muſt needs have the ſame Ratio to its circumſcribing Cylinder, as the ſquare Pyramid hath to its circumſcribing Parallelopipedon, *viz.* as 1 to 3. However, to make it yet clearer, let it be farther conſidered, That

Every right Cone is conſtituted of an infinite Series of circles, whoſe diameters do continually increaſe in arithmetical progreſſion, beginning at the Vertex, or point C, the area of its baſe AB being the greateſt term, and its perpendicular height DC the number of all the terms; therefore the area of the circle of the baſe multiplied by a third part of the Altitude DC, will be the ſum of all the Series, equal to the Solidity of the Cone, by *Lemma* III.

The Curve-Superfices of every right Cone, is equal to half the Rectangle of the Circumference of its baſe into the length of its ſide.

For the curve-ſurface of every right Cone is equal to the Sector of the circle whoſe arch BC is equal to the periphery of the baſe of the Cone, and Radius AB equal to the ſlant ſide of the Cone. Which will appear very evident, if you cut a piece of paper in the form of a ſector of a circle, as ABC, and bend both the ſides of AB and AC together till they meet, and you will find it to form a right cone.



I have omitted the Demonſtrations touching the Superficies of all the foregoing Solids, becauſe I thought it needleſs, they being all compoſed of Squares, Parallelograms, Triangles, &c. which Figures are all demonſtrated before. And if the area of all ſuch Figures as compoſe the Solid be found ſeverally, and added together, the ſum will be the ſuperficial Content of the Solid.

§ VII.

§ VII. *Of the Fruſtum of a* PYRAMID.

A FRUSTUM of a Pyramid, is a remaining Part when the Top is cut off by a Plane parallel to the Baſe. To find the ſolid Content thereof, there are ſeveral Rules.

RULE I.

To the Rectangle (or Product) of the Sides of the two Baſes add the Sum of their Squares; that Sum being multiplied into one third Part of the Fruſtum's Height, will give its Solidity, if the Baſes be ſquare.

Or thus, which is the ſame in Effect;

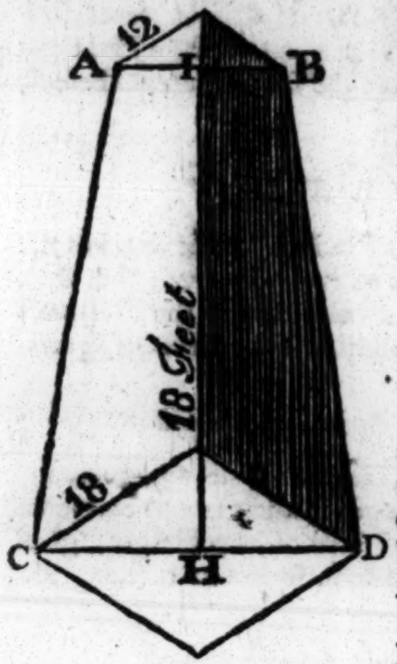
Multiply the Areas of the two Baſes together, and to the ſquare Root thereof add their two Areas; that Sum multipled by one Third of the Height, gives the Solidity of any Fruſtum, ſquare or multangled.

RULE II.

To the Rectangle of the Sides of the two Baſes add one third Part of the Square of their Difference; that Sum being multiplied into the Height will produce the Solidity, if the Baſes be ſquares: But if they be triangular or multangular, the Rectangle of the Sides with the third Part of the Square of their Difference will be the ſquare of a mean Side; and the ſquare Root thereof will be ſuch a mean Side as will reduce the tapering Solid to a Priſm equal thereunto.

Example, Let ABCD be the Fruſtum of a ſquare Pyramid, the Side of the greater Baſe 18 Inches, and the Side of the leſſer 12 Inches, and the Height 18 Feet; the Solidity thereof is required.

Firſt, multiply the two Sides together, 18 by 12, and the Product is 216, and the Difference of the Sides is 6, whoſe Square is 36; a third Part thereof is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Baſe; which multiplied by 18 Feet, the Length, the Product is 4104; this divided by 144, the Quotient is 28.5 Feet, the Content.



Or, by the firſt Rule, thus; the Square of 18 is 324, and the Square of 12 is 144, and the rectangle of 18 by 12 is 216; the ſum of theſe three 684, which multiplied by 6, the Product is 4104; which divided by 144, the Quotient is 28.5 Feet, the ſame as before.

See the Work of both Ways.

18 6 Diff.
12 6

216 3) 36 Squ.
12 add
12 a Third

228 the Sum.
18 the Height.

1824
228

144) 4104 (28.5

1224
720

...

18 12
18 12

324 Squ. 144 Squ.
144
216

684 the Sum.
6 a 3d. of the Height.

144) 4104 (28.5 Feet.

1224
720

...

By Feet and Inches thus :

	F.	I.	I.
Multiply	1	6	6
by	1		6
Product	1	6	3)36q.
add	0	1	
			12
Multiply	1	7	
by	18	0	Height.
	18	0	
	9	0	
	1	6	
Content	28	6	

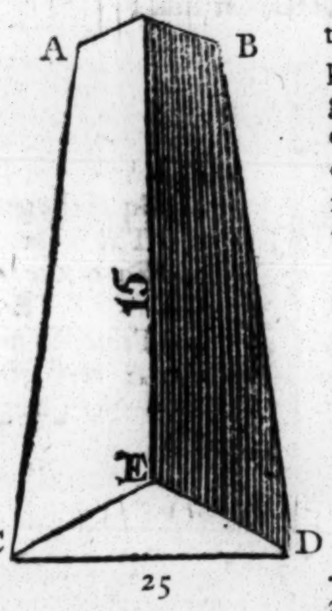
Or thus,

F.	I.	
2	3	Sq. of the greater.
1	6	The rectangle.
1	0	Sq. of the leſs.
4	9	Trip. of a mean Area.
6	0	A third of the height.
28	6	

To find the ſuperficial Content.

The perimeter of the greater baſe is 72, and the periameter of the leſſer baſe is 48 ; add both the perimeters together, the ſum is 120 ; the half thereof is 60 ; which multiplied by 18 feet, the product is 1080 ; this divided by 12, the quotent is 90 feet ; to which add the two baſes 2.25 feet, and 1 foot, the ſum is 93.25 feet, the whole ſuperficial content.

18	12	18	The height.
4	4	60	
72	48	12)1080	
48		90	Feet.
		2.25	The greater baſe
2)120		1	The leſſer baſe.
60		93.25	Sum.



Again, Let ABCD be the fruſtum of a triangular pyramid, each ſide of the greater baſe 25 inches, and each ſide of the leſſer baſe 9 inches, and the length 15 feet; the Solid Content thereof is required.

By the ſecond rule, multiply 25 by 9, and the product is 225; and the difference between 25 and 9 is 16, which ſquared makes 256, a third part thereof is 85.333, which added to 225, and the ſum is 310.333: this multiplied by .433, the product is 134.374, &c. which is the area of a mean baſe: and that multiplied by 15 feet, the length, the product is 2015.610, which divided by 144, the quotient is 13.99 feet, the ſolidity.

Or thus, by the latter part of the firſt Rule; Find the area of the greater baſe, which you will find to be 270.625, and the area of the leſſer baſe will be 35.073; theſe two areas multiplied together, the product is 9491.630625, the ſquare root thereof is 97.425, to which add the two areas, and the ſum is 403.123; which multiplied by a third part of the length 5, and the product is 2015.610; and that divided by 144, and the quotient is 13.99 feet, as before.

See the Working of both.

25 25
9 9

Product 225 16 Difference.
16

96
16

3) 256 the Square.

85.333 a third Part.
225. add.

310.333
.433 Tabular Numb. (*vide p.* 95)

930999
930999
1241332

134.374189 mean Area.
15 Length.

671870945
134374189

144)2015.61|2835(13.99 Feet.

575
1436
1401

105

25
25
125
50
625 Square.
·433
1875
1875
2500
270.625 Areas.

9
9
81 Square.

·433
81
433
3464
35·073

270.625
35·073
811875
1894375
1353125
811875

· ·
9491.630625(97 425
81
187)1391
1309
1944)8263
7776
19482)48706
38964
194845)974225
974225
······

270625

270.625 Greater area.
97.425 The mean proportional.
35.073 The leſſer area.

403.123 The triple of a mean area.
5 A third part of the height.

144)2015.615(13.99 Feet, the ſolidity.
575
1436
1401

105

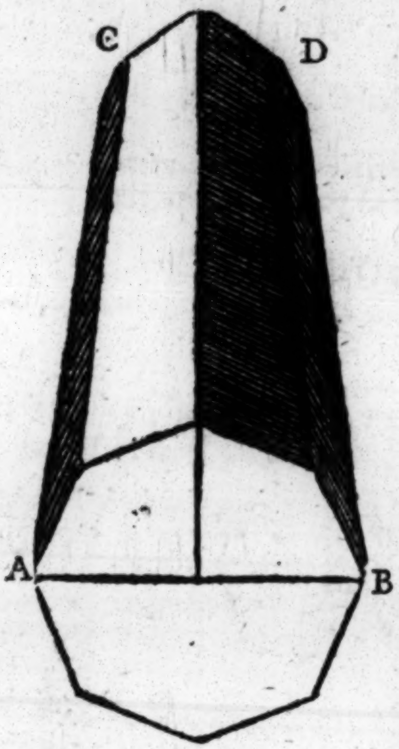
In finding the area of the triangular baſe I multiplied by .433, becauſe that is the area of the equilateral triangle when the ſide thereof is 1. A table of the areas, or multipliers, for finding the areas of Polygons, you'll find in page 95.

Multiply the Square of the Sides by the tabular Number, and the Product is the Area of the Polygon.

To find the ſuperficial Content.

The perimeter of the greater baſe is 75, and the perimeter of the leſſer baſe is 27; the ſum of both is 102, and the half ſum is 51, which multiplied by 15 feet, the product is 765; which divided by 12, the quotient is 63.75; to which add the ſum of the two baſes 2.12 feet, and the ſum is 65.87 feet, the whole ſuperficial content.

Note, That 51 ſhould have been multiplied by the ſlant height, but the difference it would make is but .06 of a foot, which is inconſiderable.



Again, Suppoſe A B C D to be the fruſtum of a Pyramid, having an Octagonal baſe, each ſide thereof being 9 inches, and each ſide of the leſſer baſe 5 inches, and the length or height 10.5 feet; the Solidity is required.

By the ſecond Rule. Multiply the greater ſide 9, by the leſſer ſide 5, and product is 45; then the difference between 9 and 5 is 4, which ſquared, makes 16; and a third part thereof is 5.3333 which added to 45, the ſum is 50.3335; multiply this laſt by the number in the table, 4.8284, and the product is 243.0292, the area of the mean baſe; which multiplied by the height 10.5 feet, and the product is 2551.8066; then divide this laſt product by 144, and the quotient is 17.72 feet, the ſolid content.

See the Work.

Mult. 9 Inches. 9 from the greater Side.
by 5 Inches. 5 ſubtract the leſſer.

Prod. 45 4
4

3) 16 ſquared.

5.3333 a third Part.
Add 45

Sum 50.3333 the Square of mean Side.
4.8284 Tabular Numb. Page 95.

2013332
402666
10067
4026
201

243.0292 a mean Area.
10.5 the Height.

12151460
2430292

144) 2551.80|660 (17.72
144

1111
1008

1038
1008

300
288

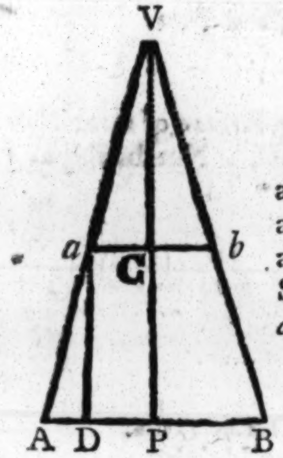
12

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 40, and their Sum is 112; the Half thereof is 56, which multiplied by the Height 10.5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet: to which add the Sum of the two Baſes 3.55, and the Sum is 5255 Feet, the whole ſuperficial Content.

DEMONSTRATION.

From the Rules delivered in the fourth and ſixth Sections, the two foregoing Rules may eaſily be demonſtrated.



Suppoſe a ſquare Pyramid, as ABV, to be cut by a Plane at *a b*, parallel to its Baſe AB, and it were required to find the Solidity of the Fruſtum or Part *a b* AB. Let there be given

$D=BA$, the Side of the greater Baſe.
$d=ba$, the Side of the leſſer Baſe.
$H=CP$, the Perpendicular Height.

Firſt, | 1 | $D-d : H :: d : \frac{dH}{D-d} = VC$ by the Fig.
Then | 2 | $DD \times \frac{H \times VC}{3} =$ the whole Pyra. BVA. by Section the fourth.
And | 3 | $dd \times \frac{1}{3}VC =$ the Pyramid aVb cut off.

Then, in the 2d and 3d Steps, if inſtead of VC you take $\frac{dH}{D-d} =$ to it by the firſt Step, it will be,

viz.

viz. 1.2	4	$\frac{DDDH}{3D-3d}$ = whole Pyramid BVA.
and 1.3.	5	$\frac{dddH}{3D-3d}$ = the Pyramid *aVb*.
4—5.	6	$\frac{DDDH-dddH}{3D-3d}$ = the Fruſtum *ab*AB

And by dividing DDD—ddd by D—d, and then multiplying the quotient by $\frac{1}{3}$ H, the laſt ſtep will be reduced to DD+Dd+dd : × $\frac{1}{3}$ H = the fruſtum *ab*AB, which in words is thus:

To the rectangle of the ſides of the two baſes, add the ſum of their ſquares; that ſum being multiplied into one third of the fruſtum's height, will give its ſolidity, which is the ſame as the firſt Rule of this ſection.

See the Work of the Diviſion.

$$\begin{array}{l} D-d)\ DDD-ddd\ (DD+Dd+dd \\ \quad\quad\ \ DDD-DDd \\ \hline \quad\quad\quad\quad\quad DDd-ddd \\ \quad\quad\quad\quad\quad DDd-Ddd \\ \hline \quad\quad\quad\quad\quad\quad\quad Ddd-ddd \\ \quad\quad\quad\quad\quad\quad\quad Ddd-ddd \\ \hline \quad\quad\quad\quad\quad\quad\quad\quad 0 \quad\quad 0 \end{array}$$

The ſame reaſon will hold good for all fruſtums of Pyramids or Cones, whether the baſe be triangular or multangular, becauſe the ſquares of the ſides of any figure, or the ſquares of the diameters of circles, are proportional to the area, which proves the latter part of the firſt ſaid Rule.

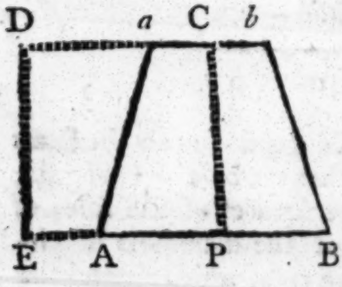
Again, to prove the ſecond Rule.

Suppoſe	1	$x=D-d$. And $F=$ the Fruſtum.
then	2	$DD+Dd+dd=\frac{3F}{H}$ by the laſt.
1⊚2	3	$xx=DD-2Dd+dd$.
2÷3	4	$3Dd=\frac{3F}{H}-xx$.
4—3	5	$Dd=\frac{F}{H}\frac{1}{3}xx$. Or $Dd+\frac{1}{3}xx=\frac{F}{H}$
5×H	6	$DD+\frac{1}{3}xx:\times H=F$, the fruſtum *ab*AB.

Which in Words is thus:

To the rectangle of the ſides of the two baſes add one third part of the ſquare of the difference of the ſaid ſides, and multiply the ſum by the height of the fruſtum, the product is the ſolidity of the fruſtum.

The ſuperficial contents of fruſtums (all but the baſes) are compoſed of Trapeziums, ſo many as the fruſtum has ſides. As the ſquare fruſtum *ab*AB, in the laſt figure, is compoſed of four Trapeziums, having the two upper and alſo the two lower angles equal; if therefore the Trapezium *ab*AB be cut in two by the line CP, and the two pieces laid together, the line *b*B upon the line *a*A, the narrow end of the one to the broad end of the other, it will form a right angled parallelogram, as is plain by the figure annexed; the parallelogram DCEP being equal to the Trapezium *ab*AB, becauſe the ſide D*a* is equal to PB, and EA, is equal to *a*C. Therefore, to find the area of the Trapezium, add half the ſides *ab* to half the ſide AB, and it makes CD or EP; which multiply by the height PC, the product is the area of the parallelogram DCEP, equal to the trapezium *ab* AB; then, if that be multiplied



tiplied by the Number of Trapeziums, the Product will be the ſuperficial Content of the Fruſtum, wanting the Baſes. Or if the whole Perimeter of the greater Baſe be added to the Perimeter of the leſſer Baſe and half the Sum multiplied by the Height, the Product will be the ſuperficial Content of all Trapeziums at once.

NOTE, That half the Sum of the Perimeters ſhould be multiplied by the ſlant Height, up the Middle of one of the Trapeziums; but in the foregoing *Examples* I have multiplied by the perpendicular Height, becauſe the Difference is very inconſiderable.

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§ VIII. *Of a Fruſtum of a* CONE.

A FRUSTUM of a CONE is that Part which remains, when the Top End is cut off by a Plane Parallel to the Baſe. To find the ſolid Content, the Rules are the ſame in Effect as for the Fruſtum of a Pyramid.

RULE I.

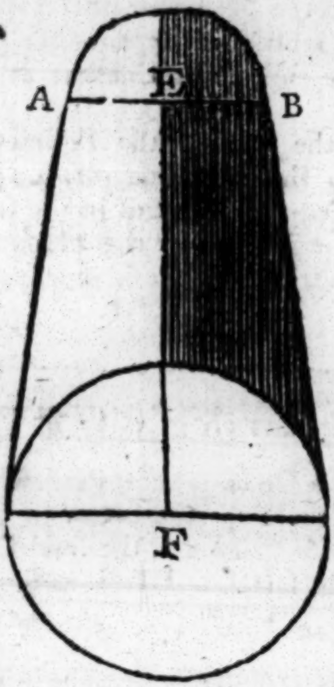
To the Rectangle of the Diameters of the two Baſes add the Squares of the ſaid Diameters, and multiply the Sum by .7854, the Product will be the Triple of a mean Area, which multiplied by $\frac{1}{3}$ of the perpendicular Height, that Product will be the ſolid Content.

Or thus: Multiply the Areas of the greater and leſſer Baſes together, and out of the Product extract the ſquare Root and add the two Areas and ſquare Root together and multiply the Sum by one Third of the perpendicular Height, the Product is the ſolid Content.

RULE II.

To the Rectangle of the greater and leſſer Diameters, add one third Part of the Square of their Difference,

ference, and multiply the Sum by .7854, the Product is a mean Area; which multiplied by the perpendicular Height, the Product is the Solidity.



Example. Let ABCD be a Fruſtum of a Cone, whoſe greater Diameter CD is 18 inches and the leſſer Diameter AB 9 inches, and the Length 14.25 Feet; the ſolid Content is required.

Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9 is 9, whoſe Square is 81, a third Part is 27; which add to 162, the Sum is 189; this multiplied by .7854 the Product is 148.44; which divided by 144, the Quotient is 1.03 Feet, the Area of a mean Baſe; which multiply by 14.25 Feet, the Height, the Product is 14.6775 Feet, the ſolid Content.

Or thus, by the firſt Rule.

The ſquare of 18 (the greater Diameter) is 324, and the Square of 9 (the leſſer Diameter) is 81, and the Rectangle, or Product of 18 by 9, is 162; the Sum of theſe three is 567, which multiplied by .7854, the Product is 445.3218; which divided by 144 the Quotient is 3.09 Feet, the triple Area of a mean Baſe; this multiplied by 4.75 Feet (a third Part of the Height) and the Product is 14.6775 Feet, the Solidity, the ſame as before.

Therefore the aggregate, or ſum, of all the rings made by the revolution of EF's, muſt be equal to that of all the circles made by the motion of GH's, *i. e.* the diſh-like ſolid, formed by the revolving rings, will be equal to the Cone formed by the revolution of the GH's, which are the elements of the triangle ABD; that is, the diſh-like ſolid will be as the Cone, $\frac{1}{3}$ of the circumſcribing Cylinder, and conſequently the hemiſphere muſt be $\frac{2}{3}$ of it: wherefore the ſphere is $\frac{2}{3}$ of the circumſcribing Cylinder.

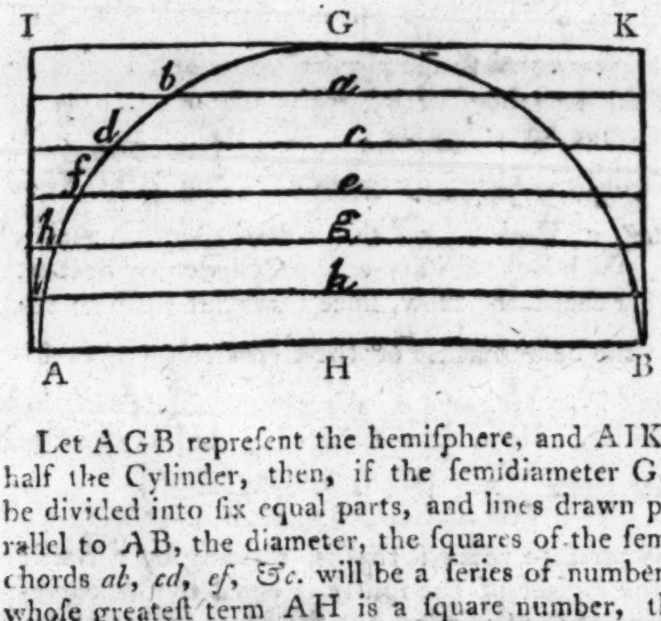
Let the radius of the ſphere be r = CD then the diameter will be 2 r; let the ſurface of the ſphere, generated by the revolving ſemicircle, be called S, and that of the Cylinder, formed by the revolution of 2 AC = 2r = Diameter, be called ſ. Wherefore, in what was juſt now proved, the expreſſion for the ſolidity of the ſphere in this notation will be $\frac{1rS}{3}$, and putting c equal to the circumference of the baſe, or for the periphery of a great circle of the ſphere, the curve-ſurface of the Cylinder will be 2 r c; alſo $\frac{rc}{2}$ will be the area of a great circle, by Sect. IX. of (Chap. 1. *Prob.* 1.) and this multiplied by 2 r, makes 1 rc, which is the ſolidity of the Cylinder, by Sect. V. of this chapter. Now, ſince ſ was put equal to 2rc = to the curve-ſurface of the Cylinder $\frac{ſr}{2}$, (by ſubſtituting ſ for 2rc) will be alſo = to the ſolidity of the Cylinder. Now, ſince the ſphere is = $\frac{2}{3}$ of the Cylinder, $\frac{rſ}{3} = \frac{2 \times ſr}{3 \times 2}$; that is, $\frac{rS}{3} = \frac{2\,ſr}{6} = \frac{ſr}{3}$.

Wherefore rS = r ſ. that is, dividing by r, S = ſ; or the ſurface of the ſphere is equal to the curve-ſurface of the Cylinder, but the curve-ſurface of the Cylinder was 2 r c.

Wherefore to find the area of the ſurface of either ſphere or Cylinder, you muſt multiply the diameter (= 2r) by the circumference of a great circle of the ſphere, or by the periphery of the baſe. From this notation alſo $\frac{r\,c}{2}$, the area of a great circle of the ſphere, is plainly $\frac{1}{4}$ of 2 r c, the ſurface of the ſphere; that is, the ſurface of the ſphere is quadruple of the area of the greateſt circle of it.

Wherefore, to 2 r c, the convex ſurface of the Cylinder, add r c equal to the area of both its baſes, you will have 3 r c; which ſhews you, that the ſurface of the Cylinder (including its baſes) is to the ſurface of the ſphere as 3 to 2; or that the ſphere is $\frac{2}{3}$ of the circumſcribing Cylinder, in area as well as ſolidity.

Or you may prove the ſphere to be $\frac{2}{3}$ of the Cylinder of the ſame baſe and altitude, by *Lemma* VI. aforegoing, thus:



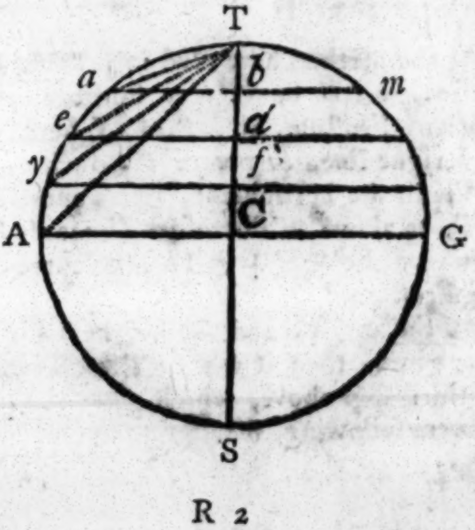
Let AGB repreſent the hemiſphere, and AIKB half the Cylinder, then, if the ſemidiameter GH be divided into ſix equal parts, and lines drawn parallel to AB, the diameter, the ſquares of the ſemichords *ab*, *cd*, *ef*, &c. will be a ſeries of numbers, whoſe greateſt term AH is a ſquare number, the others differing by odd numbers, that is, AH is 36, *kl* 35, *gb* 32, *ef* 27, *cd* 20, *ab* 11: But an infinite ſeries of ſuch numbers are in proportion to an infinite number of terms equal to the greateſt, as 2

2 to 3. And becauſe the hemiſphere is compoſed of an infinite number of circles, whoſe diameters are the chords of the ſemicircle, and the half Cylinder is compoſed of an infinite number of circles, whoſe diameters are all equal to the diameter of the ſemicircles AB; therefore the hemiſphere is in proportion to the half Cylinder as 2 to 3; and conſequently the whole ſphere bears the ſame proportion to the whole Cylinder.

That the ſuperficies of every ſphere (or globe) is equal to four times the area of its greateſt circle is thus proved.

The ſolidity of the ſphere is conſtituted of an infinite number of parallel circles, as is aforeſaid; conſequently the ſuperficies of the ſphere will be compoſed of the peripheries of thoſe circles which conſtitute its ſolidity.

NOTE, In the following demonſtrations, ⊙ ſignifies any circle in general; and if any two letters be joined to it, thus, ⊙ AB, *&c.* then it denotes the area of ſuch a circle as thoſe two letters repreſent, the radius of



Let $D = TS$, the axis of any ſphere, then, according to the property of a circle, it

will be	1	$D - Tb \times Tb = \square ab$;
that is,	2	$D \times Tb - \square Tb = \square ab$;
therefore	3	$D \times Tb = \square aT$.
For	4	$\square ab + \square Tb = \square aT$, (*Euc.* 1. 47.)
And	5	$D \times Td = \square eT$,
	6	$D \times Tf = \square yT$.

Hence it is evident, that the ſeries $\square aT$, $\square eT$, $\square yT$, *&c.* are in the ſame ratio with Tb, Td, Tf, *&c. viz.* in arithmetical progreſſion: whence it follows, that the $\odot aT =$ to the ſum of all the circle's peripheries between T and *b*.

And $\odot eT =$ the ſum of all the circle's peripheries between T and *d*, *&c.*

Conſequently, that the $\odot AT =$ the ſum of all the circle's peripheries, included between T and C; that is, $\odot AT =$ the ſuperficies of the hemiſphere.

And becauſe $\square AC + \square TC \square = AT$, and $\square AC$ is equal to $\square TC$; therefore $\odot AT = 2 \odot AC$, is the ſuperficies of the hemiſphere.

Conſequently, $4 \odot AC$ will be the ſuperficies of the whole ſphere.

SCHOLIUM.

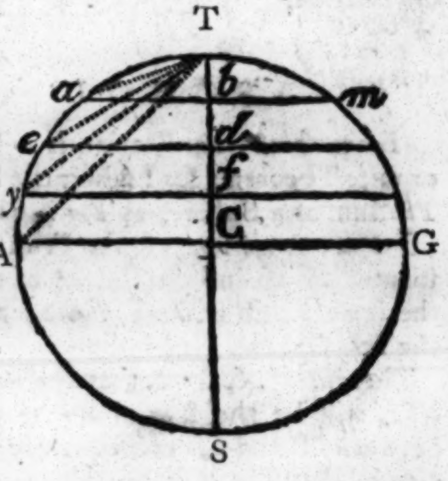
From the method here uſed in proving the whole ſuperficies, it will be eaſy to find the curve-ſuperficies of any Fruſtum, or part of a ſphere, that is cut off by a right line, or plane, *viz.* ſuch as the Fruſtum aTm in the laſt ſcheme, whoſe curve-ſuperficies is $\odot aT$, as above. Therefore (becauſe $\square ab + Tb = aT$) it will be $\odot ab + \odot Tb =$ the curve-ſuperficies of that Fruſtum.

But if the axis TS and the height Tb of the Fruſtum are given, then it will be $TS \times Tb = \square aT$, as in the third ſtep above, which gives the proportion or theorem following, *viz.*

As the axis of the ſphere : to the whole ſuperficies of the ſphere : : ſo is the height of any Fruſtum : to its curve-ſuperficies.

To which if there be added the area of the Fruſtum's baſe, the ſum will be the whole ſuperficies of the Fruſtum.

That the ſolidity of every ſphere is two thirds of its circumſcribing Cylinder, *may be thus proved.*



According to the Work above, it appears that $\odot ab$, $\odot ed$, $\odot yf$, &c. do conſtitute the Solidity of the Sphere ; and that $\square aT$, eT, yT, &c. are a ſeries of terms in arithmetical progreſſion, $\square AT$ being the greateſt term, and TC the number of terms ; therefore $\odot AT \times \frac{1}{2} TC =$ the ſum of all the Series, by *Lemma* II.

And becauſe $\square aT - \square Tb = \square ab$. $\square eT - \square Td = \square ed$. $\square yT - \square Tf = \square yf$. $\square AT - \square TC = \square AC$, &c. wherein $\square Tb$, $\square Td$, $\square Tf$, &c. are a ſeries of Squares, whoſe roots Tb, Td, Tf, are in aritmetical progreſſion ; $\square TC$ being the greateſt term, and TC the number of terms ; therefore $\odot TC \times \frac{1}{3} TC =$ the ſum of all the Series, by *Lemma* III.

Conſequently, $\odot AT \times \frac{1}{2} TC : - \odot TC \times TC =$ the ſum of all the Series $\odot ab$, $\odot ed$, $\odot yf$, &c. which conſtitute the Solidity of the half Sphere ATG. Put $D = 2TC$, the Axis of the Sphere ; then $\frac{1}{3} D = \frac{1}{2} TC$,

$\frac{1}{2}$TC, and $\frac{1}{6}$ D$=\frac{1}{3}$TC, and becauſe $\square$AT$=2\square$TC, therefore $\odot$AT$=2\odot$TC$=1.5708$ DD; and 1.5708 DD$\times\frac{1}{4}$D$=0.3927$ DDD.

Again, $\odot$TC$\times\frac{1}{3}$TC$=0.7854$ DD$\times\frac{1}{6}$D$=.1309$ DDD, then 0.3927 DDD-0.1309 DDD$=0.2618$ DDD, the Solidity of the half Sphere.

Conſequently, 0.2618 DDD$\times 2=.5236$ DDD, will be the ſolid content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the diameter of whoſe baſe, and alſo its height, is$=$D.

For 0.7854 DDD$=$the Solidity of the Cylinder, by Sect. V. But $\frac{2}{3}$ of 0.7854 DDD$=0.5236$DDD, as before.

SCHOLIUM.

From this demonſtration it will be eaſy to deduce or raiſe Theorems for finding the ſolid content of any Fruſtum of a Sphere, as T*m* in the laſt figure.

For we there ſuppoſe the Fruſtum *a*T*m* to be conſtituted of an infinite Series of circles, which have the ſame Ratio with all thoſe circles that conſtitute the half Sphere.

Therefore it follows, that $\odot a$T$\times\frac{1}{2}$Tb :—$\odot b$T$\times\frac{1}{3}$Tb, will be the ſum of all the circles intercepted between T and *b*; conſequently it will be the Solidity of that Fruſtum.

And becauſe $\square\, ab+\square$T$b=a$T, therefore $\odot ab+\odot$T$b\times\frac{1}{2}$Tb : —$\odot$T$b\times\frac{1}{3}$T$b=$the Solidity. Let c$=ab$, half the diameter of the Fruſtum's baſe; h$=$Tb its height, and S$=$ the Solidity of the Fruſtum. Then $\odot ab=3.141600$, and $\odot$T$b=3.1416$ hh; conſequently,

$$\frac{3.1416\,\mathrm{cch}+3.1416\,\mathrm{hhh}}{2}\quad\frac{3.1416\,\mathrm{hhh}}{3}=\mathrm{S};$$

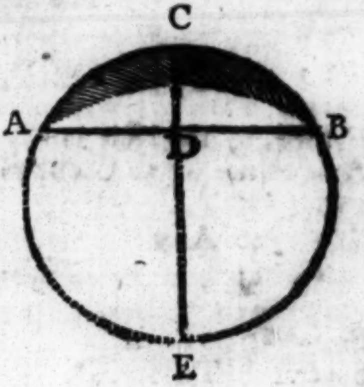
which being reduced, will become $3\mathrm{cch}+\mathrm{hhh}\times 0.5236=\mathrm{S}$: Which is one Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in words, thus:

If to three times the ſquare of the ſemidiameter of the Fruſtum's baſe you add the ſquare of the height of the Fruſtum, and multiply the ſum by the height of the Fruſtum, and that product multiplied by .5236, the product will be the ſolid content.

But if the axis of the ſphere, and the height of the Fruſtum be given, then put $D=$ the axis, $h=$ the height of the Fruſtum, and c as before: it will be $D-h\times h=cc$, *viz.* $Dh-hh=cc$. Then will $3Dhh-2hhh=3cch+hhh$; conſequently $3Dhh-2hhh\times 0.5236=S$, the Fruſtum's ſolidity; which is another theorem for finding the ſolidity of the Fruſtum, and may be expreſſed in words thus:

From three times the axis ſubtract twice the height of the Fruſtum, and multiply the remainder by the ſquare of the height, and that product multiply by .5236, this laſt product will be the ſolidity of the Fruſtum.

Example. Let ABCD be the Fruſtum of a Sphere; ſuppoſe AB (the diameter of the Fruſtum's baſe) be 16 inches, and C D (the height) 4 inches; the ſolidity is required.



By the firſt Rule.

8
8
64 Square of the Semidiameter AD.
3
192
16 Add the Square of C D.
208
4 Multiply by CD.
832

.5236
832
10478
15708
41888
The Solidity 435.6352

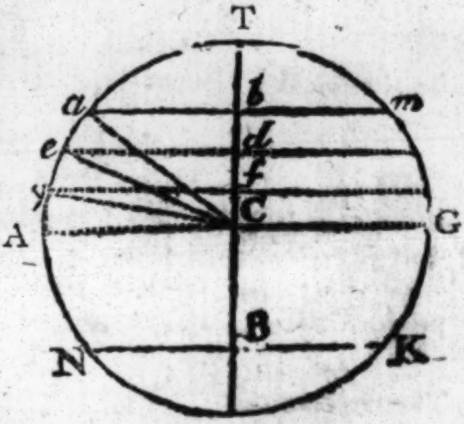
By the ſecond Rule, thus:

Firſt, by the Rule in Page 117, you will find the Axis of the whole Globe to be 20 Inches.

20 Axis
3
From 60
Subtr. 8 Twice CD.
Rem. 52
Mult. 16 Square of CD.
312
52
Prod .832

5236
832
10472
15708
41888
435.6352 The ſolid Content the ſame as before.

* And if it be required to find the middle part *am* NK, uſually called the middle Zone of a Sphere;



Then becauſe it is ſuppoſed that $am = NK$, or (which is all one) that $bC = CB$, therefore it is plain, that if twice the ſegment *a*T*m* be taken from the whole Sphere, there will remain the middle Zone *a m* NK.

But becauſe that Work is a little troubleſome, I will here ſhew how to raiſe a theorem for the doing it.

Firſt, becauſe $AC = yC = eC = aC = TC$, therefore it will be $\square AC - \square Cf = \square yf$, $\square AC - \square Cd = \square ed$, $\square AC - \square Cb = \square ab$, &c.

Here, becauſe $\square AC$, $\square AC$, $\square AC$, *&c.* are a ſeries of equals, and Cb the number of all the terms; therefore $\square AC \times Cb =$ the ſum of all that ſeries, *per Lemma* I.

And $\square Cf$, $\square Cd$, $\square Cb$, &c. being a ſeries of ſquares whoſe roots are in arithmetical progreſſion, beginning at the center C, *viz.* o. Cf, Cd, Cb, &c. wherein the greateſt term is $\square Cb$, and the number of terms is Cb; therefore $\square Cb \times \frac{1}{3} Cb =$ the ſum of all the Series, *per Lemma* III.

Conſequently, the $\odot AC \times Cb : - \odot Cb \times \frac{1}{3} Cb =$ the ſum of all the Series $\odot yf$, $\odot ed$, $\odot ab$, *&c.* which do conſtitute the ſolidity of the Half Zone *a m* AG.

And becauſe $\square AC - \square CB = \square ab$, therefore $\odot AC - \odot ab = \odot Cb$. Conſequently $\odot AC \times Cb : - \frac{\odot AC + \odot ab : \times Cb}{3} = 2 \odot AC + \odot ab . \times \frac{1}{3} Cb$ will be the ſolidity of the half Zone.

Put $D = AG = 2AC$, $x = am$, and $H = bB = 2Cb$.

Then $\odot AC = .7854DD$, $\odot ab = .7854 xx$. And if we turn the common Factor .7854 into a diviſor 1.27323, and then take the triple of that diviſor, *viz.* 3.8197, the reſult of the precedent Work will produce the following theorem.

Theor. $\left\{ \frac{2DD + xx}{3.8197} : \times H = \right\}$ the middle Zone *am* [NK.

Which in words is thus: To twice the ſquare of the axis AG, add the ſquare of the diameter of the Fruſtum's baſe, (*am*) and divide the ſum by 3.8197, then multiply the quotient by the height or thickneſs of the middle Zone, and the product will be the ſolidity of the middle Zone required.

This is ſo plain and eaſy it needs no *Example.*

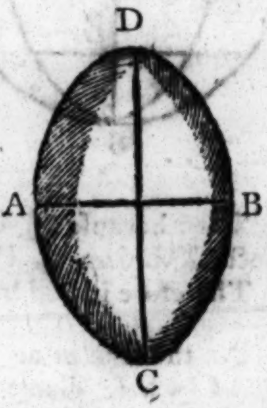
§ XII. *Of a* SPHEROID.

A SPHEROID is a Solid reſembling an Egg. To find the ſolid content thereof, this is

The RULE.

Multiply the ſquare of the diameter of the greateſt circle by the length, and that product multiply again by .5236, this laſt product will be the ſolidity of the Spheroid.

Let AB the Diameter of the greateſt Circle, be 33 Inches, and CD, the Length 55 Inches; the Solidity required.



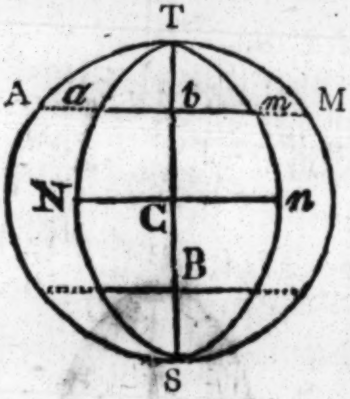
```
    33
    33
  ----
    99
   99
  ----
  1089
    55
  ----
  5445
 5445
 -----
 59895
```

```
     59895
     .5236
    ------
    359370
   179685
  119790
 299475
 ---------
 31361.0220 the Solidity
```

DEMON-

DEMONSTRATION.

Every Spheroid is equal to $\frac{2}{3}$ of Cylinder, whoſe baſe is equal to the greateſt circle of the Spheroid, and its Height equal to the Length of the Spheroid.



Suppoſe the figure NT*n* SN in the annex'd ſcheme, to repreſent a Spheroid, formed by the Rotation of the Semi-Ellipſis TNS, about its tranſverſe axis TS.

Let D=TS, the length of the Spheroid, and the axis of its circumſcribing Sphere; and d=N*n* the Diameter of the greateſt Circle of the Spheroid.

Then becauſe □ TC : NC : : □ A*b* : □ *ab*, by Sect. XV. *Step.* 3. *Page* 127.

Therefore it will be DD : dd : : □ A*b* : □*ab*.

But the ſum of an infinite ſeries of ſuch circles as ⊙ A*b* [whoſe diameters are Chords] do conſtitute the Solidity of the Sphere [by Sec. XI.]

And the Sum of an infinite ſeries of ſuch circles as ⊙*ab*, [*viz.* whoſe Diameters are Ordinates of the Ellipſis] do conſtitute the Solidity of the Spheroid.

Therefore DD : dd :: 0.5236DDD : 0.5236Ddd =the Solidity of the Spheroid, [*Eucl.* 5. 12.]

But 0.5236 Ddd=$\frac{2}{3}$ of the Cylinder, whoſe diameter is=d, and height=D. [by Sect. V.]

Now from this proportion between the ſphere and its inſcribed Spheroid, it will be very eaſy to deduce Theorems for finding the ſolid content either of the Fruſtum or middle Zone of any Spheroid, having the ſame height with that of the ſphere; for,

See the WORK.

18
9
162
Add 27
Sum 189

18 from
9 ſubtract
9 remains.
9
3) 81 Square.
27 a Third.

.7854
189
70686
62832
7854
144)148.44|06(1.03
144
444
432
12

Height 14.25 Feet.
Area Baſe 1.03 Feet.
4275
14250
Solid Content 14.6775 Feet.

324 The Square of 18.
162 The Rectangle.
81 The Square of 9.
567 The triple Square of a mean Diameter.

.7854
567
54978
47124
39270
144) 445.32|18 (3.09
1332
36

4.75 a third of the Height.
1545
2163
1236
The Solidity 14.6775

To find the ſuperficial Content.

By Chap. I. Sect. IX. *Problem* 2. you will find the circumference of the greater baſe to be 56.5488, and of the leſſer baſe 28.2744; the Sum of both is 84.8232; the half Sum is 42.4116; which multiplied by 14.25 Feet, and the Product is 604.36, *&c.* which divided by 12, the Quotient is 50.36 Feet the curve-ſurface; to which add the Sum of the two baſes, 2.21 Feet, the Sum is 52.57 Feet, the whole ſuperficial Content.

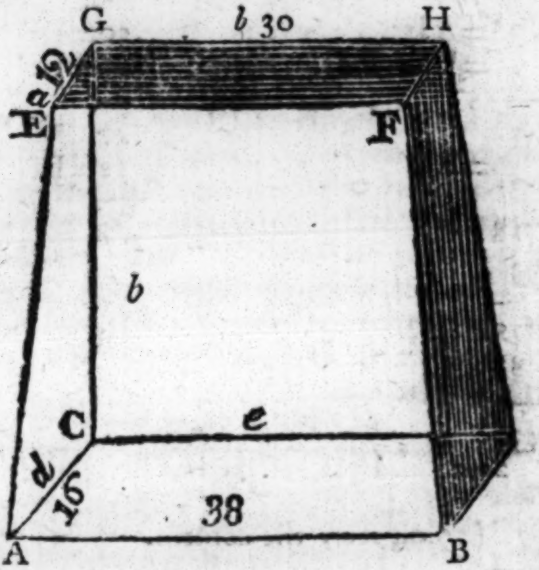
§ IX. *To meaſure the* Fruſtum *of a rectangled* Pyramid, *called a* PRISMOID, *whoſe Baſes are parallel to one another, but diſproportional.*

The RULE.

TO the greateſt length add half the leſſer length, and multiply the ſum by the breadth of the greater baſe, and reſerve the Product.

Then, to the leſſer length add half the greater length, and multiply the ſum by the breadth of the leſſer baſe, and add this product to the other product reſerved, and multiply that Sum by a third Part of the height, and the product is the ſolid Content.

Example.



Example. Let ABCDEFGH be a Priſmoid given, the length of the greater baſe AB 38 inches, and its breadth AC 16 inches; and the length of the leſſer baſe EF is 30 inches; and its breadth 12 inches, and the height 6 feet; the ſolid content is required.

To the greater length AB 38, add half EF the leſſer length 15, the ſum is 53; which multiply by 16, the greater breadth and the product is 848, which reſerve.

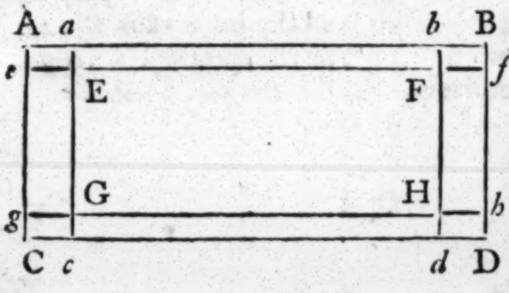
Again, To EF 30, add half AB 19, and the ſum is 49; which multiply by 12 (the leſſer breadth EG) the product is 588; to which add 848, (the reſerved product) and the ſum is 1436; which multiply by 2, (a third part of the height) and the product is 2872; divide this product by 144, and the quotient is 19.94 feet, the ſolid content.

38=AB
15=Half EF
53
16=AC
318
53
848
588
1436
2=a third part of the height.
144)2872(19.94 feet, the content.
1432
1360
640
64

30=EF
19=Half AB
49
12=EG
588

To prove this RULE.

Let us ſuppoſe the ſolid cut into pieces, ſo as to make it capable of being meaſured by the foregoing Rules, thus: Let ABCD repreſent the greater baſe, and EFGH the leſſer baſe; and let the ſolid be ſuppoſed to be cut thro' by the lines *ac*, *bd*, and *ef*, *gh*, from the top to the bottom; ſo will there be a parallelopipedon having its baſes equal to the leſſer baſe EFGH, and its height 6 feet equal to the height of the ſolid: — Multiply 30, the length of the baſe by 12 (the breadth thereof,) and the product is 360; which multiplied by the height 6



feet, and the product is 2160. Then there are two Wedge-like pieces, whoſe baſes are *ab*EF, and GH*cd*; if theſe two pieces be laid together, the thick end of one to the thin end of the other, they will compoſe a rectangled Parallelopipedon; which to meaſure, multiply the length of the baſe 30, by its breadth 2, and the product is 60; which multiplied by 6 (the height) the product is 360. Then there are two other wedge-like pieces, whoſe baſes are *e*E*g*G, and F*f*H*h*; theſe two laid together will compoſe a rectangled Parallelopipedon; to meaſure this, multiply the length of the baſe 12, by the breadth 4, the product is 48; which multiplied by 6, (the height) the product is 288. And *laſtly*, there are four rectangled Pyramids, at each corner one; which to meaſure, multiply the length of one of the baſes 4, by its breadth 2, the product is 8; which multiplied by 2, (a third part of the height) the product is 16, and that multiplied by 4, (becauſe there are four of them) the product is 64. Then add all theſe together, and the Sum is 2872; and divide by 144, the Quotient is 19.94 feet, the ſame as before; which ſhews the Rule to be true.

See the Work.

12	30	12	4
30	2	4	2
360	60	48	8
6	6	6	2
2160	360	288	16
360			4
288			
64			64

144) 2872 (19.94 Feet, the whole Content.

1432
1360
640

64

To find the ſuperficial Content.

Half the Perimeter of the greater baſe is 54, and half the Perimeter of the leſſer baſe is 42, which added together the ſum is 96, which multiplied by 6, (the height) the product is 576; divide this product by 12, the Quotient is 48 feet; to which add the ſum of the two baſes 6.72 feet, and the ſum is 54.72 feet, the whole ſuperficial Content.

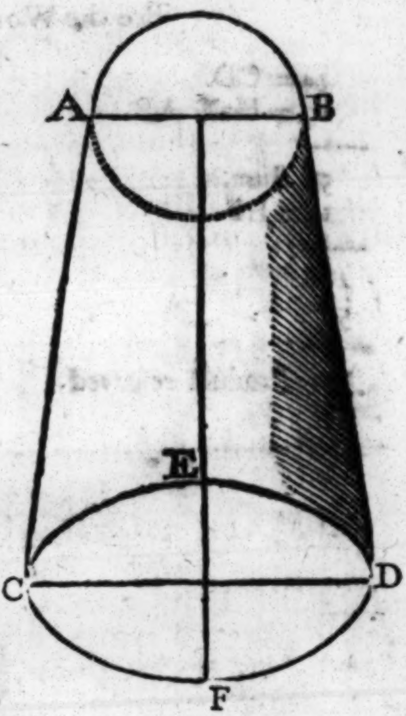
§ X. *To meaſure a* CYLINDROID; *that is, a* Fruſtum *of a* CONE *having its Baſes parallel to each other, but unlike.*

The RULE.

TO the longeſt diameter of the greater baſe add half the longeſt diameter of the leſſer baſe, and multiply the ſum by the ſhorteſt diameter of the greater baſe, and reſerve the product.

Then, to the longeſt diameter of the leſſer baſe add half the longeſt diameter of the greater baſe, and multiply the ſum by the ſhorteſt diameter of the leſſer baſe, and add the product to the former reſerved ſum, and that ſum will be the triple ſquare of a mean diameter; which multiplied by .7854, and that product multiplied by a third part of the height, the product is the ſolid Content.

Example.



Example. Let AB CD be a Cylindroid, whoſe bottom baſe is an Oval, the tranſverſe Diameter being 44 inches; and the conjugate Diameter 14 inches; and the upper baſe is a circle whoſe Diameter is 26 inches, and the height of the Fruſtum is 9 feet; the ſolidity is required.

To 44 (the greater Diameter of the lower baſe) add 13, half the Diameter of the leſſer baſe, the ſum is 57; which multiplied by 14 (the conjugate Diameter of the greater baſe, the product is 798, which reſerve. Then to 26 (the Diameter of the leſſer baſe) add 22 (half the tranſverſe Diameter of the greater baſe) and the ſum is 48; which multiplied by 26 (the Diameter of the leſſer baſe) the product is 1248; to which add the former reſerved, the ſum is 20.46; which multiplied by .7854 (the product is 1606.9284; which multiplied by 3, a third part of the height) the product is 4820.7852; which divided by 144, the quotient is 33.47 feet, the ſolid Content.

See the Work.

44=CD.
13=Half AB.
57 Sum.
14=EF.
288
57
798 Product reſerved.

26=AB.
22=Half CD.
48 Sum.
26=AB.
288
96
1248
798 Add.
2046
.7854
8184
10230
16368
14322
1606.9284
3
144) 4820.78|52 (33.47
500
687
1118
110

This Rule being the ſame with that in the laſt ſection, the proof of that may ſerve as a ſufficient proof of this, if what has been before written be well conſidered.

To find the ſuperficial Content.

To the periphery of the Ellipſis 97.41 add the periphery of the circle 81.68, and the ſum is 179.09; the half thereof 89.545, multiplied by 9, the product is 805.905; which divided by 12, the quotient is 67.16 feet; the curve-ſurface: then the area of the Ellipſis is 3.36 feet, and the area of the circle is 3.68 feet; both which added to the curve-ſurface, the ſum is 74.2 feet, the whole ſuperficial content.

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§ XI. *Of a* SPHERE, *or* GLOBE.

A Sphere, or Globe, is a round ſolid body, every part of whoſe ſurface is equally diſtant from a point within it, called its center; and it may be conceived to be formed by the revolution of a ſemicircle round its diameter. To find its ſolidity, this is

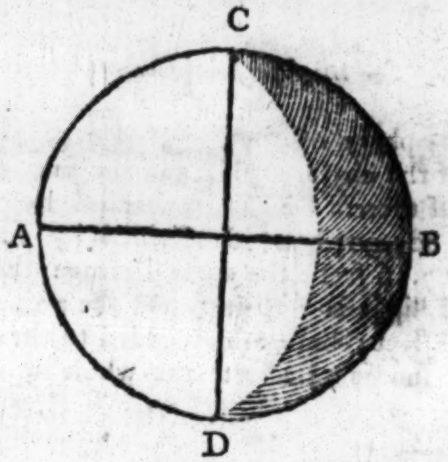
The RULE.

1. Multiply the axis or diameter into the circumference, the product is the ſuperficial content; which, multiply by a ſixth part of the axis, the product is the ſolidity.

2. Or thus: As 21 : 11 : : ſo is the cube of the axis to the ſolid content.

3. Or, as 1 : is to .5236 : : ſo is the cube of the axis to the ſolid content.

Example.



Example. Let ABCD be a Globe whoſe axis is 20 inches, then the circumference will be 62.832: Then by the firſt Rule, multiply the circumference by the axis, and the product will be 1256.64, which is the ſuperficial content in inches; take a ſixth part thereof, which is 209.44, (becauſe an exact ſixth part of 20 cannot be taken) multiply that ſixth part by 20, (the axis) and the product is 4188.8, the ſolidity in inches. Or, if you multiply the ſuperficial content by the axis, and take a ſixth part of the product, the anſwer will be the ſame.

Or thus, by the ſecond Rule:

The cube of the axis is 8000, which multiplied by 11, the product is 88000: which divided by 21, the quotient is 4190.47, the ſolidity.

Or, by the third Rule:

If the cube of the axis be multiplied by .5236, the product is 4188.8 the ſolidity, the ſame as by the firſt way. If you divide 4188.8 by 1728, the quotient is 2.424 feet.

See the WORK.

62.832

62.832
20

6) 1256.640 the ſuperficial Content.

209.44 A ſixth Part.
20

4188.80 The Solidity in Inches.

As 21 : 11 : : 8000
11

21) 88000 (4190.47 The Content.

40
190
100
160

13

As 1 : .5236 : : 8000
8000

1728) 4188.8000 (2.424 Feet, the Solidity.

7328
4160
7040

128

NOTE. If the Axis of a Globe 1, the Solidity will be .5236; if the Circumference be 1, the Solidity will be .016887.

By Scale *and* Compaſſes.

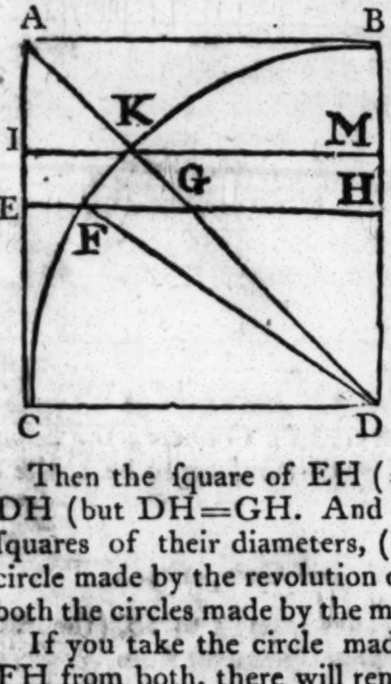
Extend the compaſſes from 1 to 20, (the Axis) that extent turned three times over from .5236, will at the laſt fall upon 4188.8, the ſolid Content in inches. Or, extend the compaſſes from 1728 to 8000 (the cube of the Axis) that extent will reach from .5236 to 2.424, the ſolid content in Feet.

Extend the compaſſes from 1 to 20, (the Axis) that extent turned twice over from 3.1416, will at laſt fall upon 1256.64, the ſuperficial content in inches. Or, extend the compaſſes from 144 to 400, (the ſquare of the Axis) that extent will reach from 3.1416 to 8.72, the ſupererficial content in feet.

DEMONSTRATION.

Every ſphere is equal to a cone whoſe perpendicular Axis is the Radius of the ſphere, and its baſe a plane, equal to all the ſurface of it.

For you may conceive the ſphere to conſiſt of an infinite number of cones, whoſe baſes, taken all together, compoſe the ſurface, and whoſe vertexes meet altogether in the center of the ſphere: Hence the ſolidity of the ſphere will be gained by multiplying its ſurface by $\frac{1}{3}$ of its Radius.



Let the ſquare AB CD, the Quadrant CBD, and the right-angled Triangle AB D be ſuppoſed all three to revolve round the line BD as an Axis; then will the ſquare generate a Cylinder, the Quadrant an Hemiſphere, and the Triangle a cone, all of the ſame baſe and altitude

Then the ſquare of EH ($=\square FD=\square FH+\square DH$ (but $DH=GH$. And ſince circles are as the ſquares of their diameters, (by *Euclid*, 12. 2.) the circle made by the revolution of EH muſt be equal to both the circles made by the motions of FH and GH.

If you take the circle made by the revolution of FH from both, there will remain the circle made by the motion of GH, equal to the Ring deſcribed by the motion of EF, and thus it will always be, whereever you draw the line EH or IM, or &c.

Therefore

As the ſolidity of the whole ſphere : is to the ſolidity of the whole Spheroid : : ſo is any part of the ſphere, to the like part of the Spheroid.

As for inſtance, ſuppoſe it was required to find the middle Zone of any Spheroid.

Let $D=TS$ and $d=Nn$ as above, and $H=bB$. $x=AM$, and $c=am$.

Then, $\left\{\frac{2DD+xx}{3.8197}\right.\times H$ = the middle Zone of the Sphere.

And $0.5236\,DDD : 0.5236\,ddD :: \frac{2DD+xx}{3.8197}\times H : \frac{2ddH}{3.8197}+\frac{xxddH}{3.8197\,DD}$ = the middle Zone of the Spheroid.

Again, $DD : dd :: xx : cc$. Therefore $\frac{xxd}{DD}=cc$.

Conſequently, $\frac{xxdd}{DD}\times\frac{H}{3.8197}=\frac{cc.}{3.8197}\times H$. Which being taken inſtead of $\frac{xxddH}{3.8197DD}$, there will ariſe This following Theorem, $\left\{\frac{2dd+cc}{3.8197}\right. : \times H$ = the middle Zone of the Spheroid.

NOTE, That $3.8197=1.2732\times3$. *See* p. 109.

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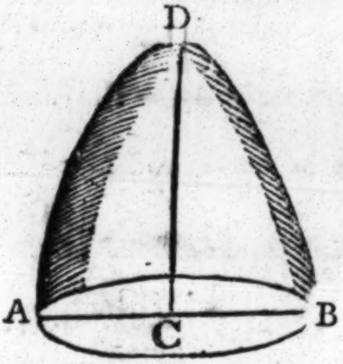
§ XIII. *Of a Parabolic* CONOID.

A PARABOLIC Conoid is ſomething like an half Spheroid, having its ſides ſomewhat ſtraighter. It is generated by ſuppoſing a Semi-Parabola turned about its axis. To find the ſolid Content thereof, this is

The RULE.

Multiply the ſquare of the diameter of its baſe by .7854, and multiply that product by half the height that laſt product ſhall be the ſolid Content.



Let ABCD be a Parabolic Conoid, the diameter of whoſe baſe is 36 inches and its height CD 33 inches; the ſolidity is required.

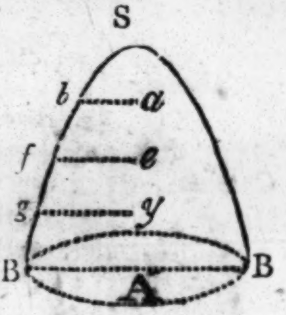
```
  36          .7854        1017.8784
  36           1296               33
----         ------        ---------
 216          47124         30536352
108          70686         30536352
----         15708         ---------
1296         7854       2) 33589.9872
             ---------     ---------
             1017.8784     16794.9936

1728) 16794.99|36 (9.71 Feet, the Content.
      15552
      -----
       12429
       12096
       -----
        3339
        1728
        ----
        1611
```

DEMONSTRATION.

The Parabolic Conoid is conſtituted of an infinite number of circles, whoſe diameters are the ordinate of the Parabola. Now, according to the property of every Parabola, it will be, SA : AB : : AB : $\frac{\square AB}{SA}$ = L, the *Latus Rectum.*

Then



$$\text{Then}\left\{\begin{array}{l} Sa\times L=\square ba, \\ Se\times L=\square fe, \\ Sy\times L=\square gy, \text{ \&c.} \end{array}\right.$$

Here $Sa\times L$, $Se\times L$, $Sy\times L$, *&c.* are a ſeries of terms in arithmetical progreſſion.

Therefore $\square\, b\, a$, $\square\, fe$, $\square\, g\, y$, &c. are alſo a ſeries of terms in the ſame progreſſion, beginning at the point S, wherein $\square$ A B is the greateſt term, and SA the number of all the terms. Therefore $\square AB\times\frac{1}{2}SA=$ the ſum of all the ſeries, (by *Lemma* II.)

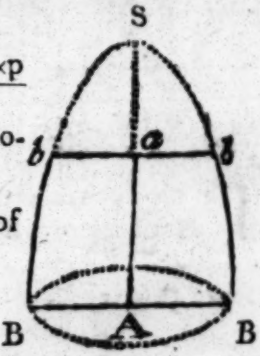
Conſequently, $\odot AB\times\frac{1}{2}SA=$ the ſum of all the ſeries of $\odot ba$, $\odot fe$, $\odot gy$, *&c.* which do conſtitute the ſolidity of the Conoid.

Put $D=2AB$ and $H=SA$.

Then $.7854\ DD\times\frac{1}{2}H=.3927DDH$ will be the ſolid content of the Conoid, which is juſt half the Cylinder whoſe baſe is $=D$, and height $=H$.

This being rightly underſtood, it will be eaſy to raiſe a theorem for finding the lower Fruſtum of any Parabolic Conoid.

For ſuppoſing $h=aA$, the height of the Fruſtum, and $p=Sa$, the height of the part bSb cut off; then $h+p=SA$, the height of the whole Conoid.



Conſequently $\frac{\odot AB\times h+\odot AB\times p}{2}$ $=$ the ſolidity of the whole Conoid.

And $\frac{\odot ba\times p}{2}=$ the ſolidity of the part cut off.

Therefore	1	$\dfrac{\odot AB\times d+\odot AB\times p-\odot ba\times p.}{2}$ the ſolidity of the Fruſtum.
But	2	$h+p:\square AB::p:\square ba.$
Conſequently	3	$h+p:\odot AB::p:\odot ba.$
$3 \because$	4	$\odot AB\times p=\odot ba\times h+\odot ba\times p.$
$4-\odot ba\times p$	5	$\odot AB\times p:-\odot ba\times p=\odot ba\times h.$
1×2	6	$\odot AB\times h:+\odot AB\times p:-\odot ba\times p. =2F.$
$6-5$	7	$\odot AB\times h=2F:-\odot ba\times h.$
$7+\odot ba\times h$	8	$\odot AB\times h:+\odot ba\times h=2F.$
$8\div 2$	9	$\dfrac{\odot AB+\odot ba}{2}:\times h=F$, the Fruſtum's Solidity.

Let $D=2AB$, as before, and $d=2ba$, the diameter of the part cut off, then we ſhall have this following Theorem.

$0.3927DD+0.3927dd:\times h=$ the Solidity of the Fruſtum required. Which in words is thus:

Multiply the ſum of the ſquares of the greater and leſſer diameters by .3927, and that product by the height of the Fruſtum, the laſt product ſhall be the ſolid Content.

---

§ XIV. *Of a Parabolic* SPINDLE.

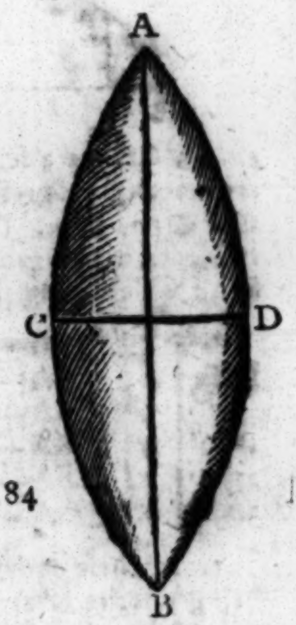
IF an accute Parabola be ſuppoſed to be moved about its greateſt ordinate, it will form a Solid called a Parabolic Spindle. To find the ſolid Content, this is

The R U L E.

Multiply the ſquare of the diameter of its greateſt circle by .4188 (being $\frac{8}{15}$ of .7854) and that product by its length, that laſt product is the ſolid content.

Let ABCD be a Parabolic Spindle, whoſe greateſt diameter CD is 36 inches, and its length AB 99 inches; the ſolidity is required.

```
  36=CD          .41888
  36               1296
 ----            ------
  216            251328
 108            376992
 ----           83776
 1296 ſqu.     41888
               --------
               54286848
                     99
               ---------
              488581632
             488581632
          ----------------
1728)53743.97952(31.10184
      ------ ......
        1903
         1759
          3179
           14515
              6912
        ---------
             ....
```

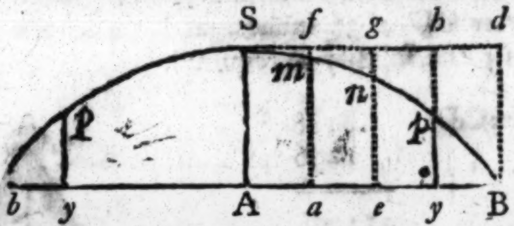


The ſolid content is 31.10184 feet.

DEMONSTRATION.

A Parabolic Spindle is conſtituted of an infinite ſeries of circles, whoſe diameters are all parallel to the axis of a Parabola, as ⊙*ma*, ⊙*ne*, ⊙*py*, &c.

Let us ſuppoſe the line S*d* parallel to AB, *&c.* then it hath already been proved, that the lines *fm*, *gn*, *hp*, &c. are a ſeries of ſquares whoſe roots are in arithmetical progreſſion ; conſequently their ſquares *viz.* $\square fm$, $\square gn$, $\square hp$, *&c.* will be a ſeries of Biquadrats, whoſe roots will be in arithmetical progreſſion : which being premiſed, we may proceed thus :



Firſt,	1	$SA - fm = ma.$
	2	$SA - gn = ne.$
	3	$SA - hp = py.$
1⊙2	4	$\square SA - 2SA \times fm + \square fm = \square ma.$
2⊙2	5	$\square SA - 2SA \times gn + \square gn = \square ne.$
3⊙2	6	$\square SA - 2SA \times hp + \square hp = \square py,$ *&c.*

1. In theſe equations, the $\square$SA, $\square$SA, $\square$SA, being a ſeries of equals, and AB the number of all the terms ; therefore it will be $\square SA \times AB =$ the ſum of all the ſeries, by *Lemma* I.

2. Becauſe *fm*, *gn*, *hp*, &c. are a ſeries of ſquares wherein SA is the greateſt term, and AB the number of all the terms ;

therefore $\frac{2SA \times SA \times AB}{3} = \frac{2\square SA \times AB}{3}$ will be the ſum of all the ſeries, by *Lemma* III.

3. And the $\square fm$, $\square gn$, $\square hp$, &c. will be a ſeries of terms in the ratio of Biquadrats, as above, $\square SA$ being the greateſt term, and AB the number of all the terms; therefore it will be $\frac{\square SA \times AB}{5}$ = the ſum of all the ſeries, by *Lemma* V.

Whence is follows, that $\square SA \times AB - \frac{2\square SA \times AB}{3} + \frac{\square SA \times AB}{5}$ = the ſum of all the ſeries of $\square ma$, $\square ne$, $\square py$, &c.

That is, $\frac{8\square SA \times AB}{15}$ = the ſum of all the ſeries, $\square ma$, $\square ne$, $\square py$, &c. Conſequently, $\frac{8\odot SA \times AB}{15}$ = the ſum of all the ſeries of circles. $\odot ma$, $\odot ne$, $\odot py$, &c. which conſtitute the ſolidity of half the Spindle, *viz.* of SAB.

Therefore putting D = 2SA, and H = 2AB, it will be 0.41888 DDH = the ſolidity of the whole Parabolic Spindle *b*SB, being $\frac{8}{15}$ of 0.7854 DDH, the ſolidity of its circumſcribing cylinder.

From hence we may alſo raiſe a theorem for finding the fruſtum SA*py* of the laſt figure.

For $\odot SA$ being the greateſt term, $\odot py$ the leaſt term, and A*y* the number of all the terms or circles included between A and *y*.

There-

Therefore,	1	$\left\{ \square SA - \frac{2SA \times hp}{3} + \frac{\square hp}{5} : \times Ay = z \right.$ the ſum of all the ſeries, $\square SA$, $\square ma$, $\square ne$, $\square py$.
$1 \times \overline{3}$	2	$3\square SA - 3SA \times hp + \frac{3\square hp}{5} : \times Ay = 3z.$
$2 - Ay$	3	$3\square SA - 2SA \times hp + \frac{3\square hp}{5} = \frac{3z}{Ay}$
But	4	$\square SA - 2SA \times hp = \square py - \square hp$, *per Step* 6
$3 - 4$	5	$2\square SA + \frac{3\square hp}{5} + \frac{3z}{Ay} - \square py + \square hp.$
5 + &c.	6	$2\square SA + \square py - \frac{2}{5}\square hp = \frac{3z}{Ay}$
Conſeq.	7	$2\odot SA + \odot py - \frac{2}{5}\odot hp : \times \frac{1}{3} Ay = z$, the ſum of all the ſeries of $\odot SA$, $\odot ma$, $\odot ne$, $\odot py$, which do conſtitute the ſolidity of the fruſtum SA *py*.

Therefore putting $D = 2SA$, as before, $C = 2py$, $x = 2hp$, and $H = Ay$, it will be $1.5708\, DD + .7854\, CC - .31416\, xx : \times \frac{1}{3} H =$ the fruſtum SA*py*. And if we make $L = 2H$, then $1.5708\, DD + .7854\, CC - .31416\, xx : \times \frac{1}{3} L =$ the double of that fruſtum, being the middle Zone: Which in words is thus:

Multiply the ſquare of the greateſt diameter by 1.5708, and multiply the ſquare of the leſſer diameter by .7854, and multiply the ſquare of the difference of the diameters by .31416; from the ſum of the two former products ſubtract the latter product, and multiply the remainder by one third part of the length, and that product will be the ſolidity of the middle Zone required.

C H A P. III.

Of Meaſuring of the Works of the ſeveral Artificers relating to Building; and what Methods and Cuſtoms are obſerved therein.

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§ I. *Of* CARPENTERS WORK.

THE Carpenters Work which are meaſurable, are, Flooring, Partitioning and Roofing; all which are meaſured by the ſquare of 10 feet long and 10 feet broad; ſo that one ſquare contains 100 ſquare feet.

1. *Of* Flooring.

If a Floor be 57 feet 3 inches long, and 28 feet 6 inches broad, how many ſquares of Flooring are there in the Room?

Multiply 57 feet 3 inches by 28 feet 6 inches, and the product is 1631 feet, *&c.* which divide by 100 (this is done by cutting off from the product two figures towards the right hand with a daſh of the pen) the remaining figures are the quotient, and the figures cut off are feet. Thus, 1631 divided by 100, by cutting off 31 from the right hand thereof the quotient is 16 ſquares, and 31 cut off is 1 feet.

See the Work both by decimals, and alſo by feet and inches.

	F.	I.	
57.21	57	3	
28.5	28	6	
28625	456		
45800	114		
11450	28	7	6
	7	0	0
16|31.625			
	16|31	7	6

Facit 16 Squares and 31 Feet.

NOTE, That, .5 is the decimal for a half of any thing, .25 is the decimal for a quarter; and 125 is the decimal for half a quarter; ſo, in the laſt *Example*, .25 is the decimal of 3 inches, becauſe 3 inches is a quarter of a foot; and 5 is the decimal of 6 inches, becauſe 6 inches is half a foot.

Example 2. Let a Floor be 53 feet 6 inches long, and 47 feet 9 inches broad, how many Squares are contained in that Floor?

	F.	I.	
47.75	53	6	
53.5	47	9	
23875	371		
14325	212		
23875	26	9	
	13	4	6
25|54.625	23	6	
	25|54	7	6

Facit 25 Squares and 54 Feet.

By Scale *and* Compaſſes.

In the firſt *Example* extend the Compaſſes from 1 to 28.5, that extent will reach from 57.25 to 16 ſquares and near a third part.

In the ſecond *Example*, extend the Compaſſes from 1 to 47.75, that extent will reach from 53.5 to 25 ſquares and above an half.

2. *Of*

2. *Of* Partitioning.

Example 1. If a Partition between rooms be in length 82 feet 6 inches, and in height 12 feet 3 inches, how many ſquares are contained therein?

The length and breadth being multiplied together, the product is 1010.625; which divided by 100, (as before is ſhewn) and the anſwer is 10 ſquares 10 feet; the inches or parts in theſe caſes are of no value.

12.25	F.	I.	
82.5	82	6	
	12	3	
6125			
2450	990	0	
9800	20	7	6
10|10.625	10|10	7	6

Facit 10 Squares 10 Feet.

Example 2. If a Partition between rooms be in length 91 feet 9 inches, and in breadth 11 feet 3 inches, how many ſquares are contained therein?

The length and breadth being multiplied together, the product it 1032 feet; which divided by 100, the anſwer will be 10 ſquares and 32 feet.

91.75	F.	I.	
11.25	91	9	
	11	3	
45875			
18350	1009	3	
9175	22	11	3
9175			
	10|32	2	3
10 32.1875			

3. *Of* Roofing.

It is a rule amongſt workmen, that the flat of any houſe and half the flat thereof taken within the walls, is equal to the meaſure of the roof of the ſame houſe; but this is when the roof is true pitched. For if the roof be more flat or ſteep than the true pitch, it will meaſure to more or leſs accordingly.

Example 1. If a houſe within the walls be 44 feet 6 inches long, and 18 feet 3 inches broad, how many ſquares of roofing will cover that houſe?

Multiply the length and breadth together, and the product is 812 feet, the flat; the half thereof is 406 feet, which added to the flat, the ſum is 1218 feet; which divided by 100, the anſwer is 12 ſquares and 18 feet.

18.25
44.5
9125
7300
7300
Flat. 812.125
Half. 406
12|18

	F.	I.	
	44	6	
	18	3	
	352		
	44		
	11	1	6
	9	0	0
The Flat.	812	1	6
The Half.	406		
Sum.	12|18		

Facit 12 Squares 18 Feet.

By Scale *and* Compaſſes.

In the firſt *Example* of Partitioning, extend the compaſſes from 1 to 12.25, that extent will reach from 82.5 to 10 ſquares and one tenth.

In the ſecond *Example*, extend the compaſſes from 1 to 11.25, that extent will reach from 91.75 to 10 ſquares, and a little leſs than a third part.

In the *Example* of Roofing, extend the Compaſſes from 1 to 18.25, that extent will reach from 44.5 to 812, the flat; to which add the half thereof, and the ſum is 12.18, which is 12 Squares 18 Feet, as above.

There are other Works about a Building, done by the Carpenter, that are meaſured by the foot, running meaſure, that is, by the number of feet in length only; as Cornices, Doors and Caſes, Window Frames, Guttering, Lintels, Sommers, Skirt boards, &c.

NOTE 1. In meaſuring of Flooring, after you have meaſured the whole Floor, you muſt deduct out of it the Well-holes for the Stairs and Chimneys; and in Partitioning, for the Doors, Windows, &c. except (by agreement) they are to be included.

NOTE 2. In meaſuring of Roofing, ſeldom any deductions are made for the holes for the Chimney Shafts, the vacancies for Lutheren-Lights and Sky-Lights; for they are more trouble to the Workman, than the Stuff which would cover them is worth.

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§ II. *Of* BRICKLAYERS WORK.

THE principal is Tiling, Walling, and Chimney-Work.

1. *Of* Tiling.

Tiling is meaſured by the ſquare of 10 feet, as Flooring, Partitioning and Roofing were in the Carpenter's Work; ſo that between the Roofing and Tiling, the difference will not be much, yet the Tiling will be the moſt; for the Bricklayers ſometimes will require to have double meaſure for Hips and Vallies. When Gutters are allowed double meaſure, the way is to meaſure the length along the ridge Tile, and by that means the meaſure of the Gutters becomes double; it is uſual alſo to allow double meaſure at the Eaves, ſo much as the projecture is over the Plate, which is commonly about 12 or 20 inches.

Example 1. There is a Roof covered with Tiles, whoſe depth on both ſides (with the uſual allowance at the Eaves) is 37 feet 3 inches, and the length 45 feet; I demand how many Squares of Tiling are contained therein?

F.	I.	
37	3	37·25
45	0	45
185		18625
148		14900
11	3	
		16|76.25
16|76	3	

Anſwer. 16 Squares 76 Feet.

Example 2. There is a Roof covered with Tiles whoſe depth on both ſides (with the allowance at the Eaves) is 35 feet 9 inches, and the length 43 feet 6 inches; I demand how many Squares of Tiling are in the Roof?

F.	I.		
43	6		35 75
35	9		43.5
215			17875
129			10725
21	9		14300
10	10	6	
17	6		15|55.125
15|55	1	6	

Here the length and depth being multiplied together, the product is 1555 feet; which divided by 100, (as before is taught) the anſwer is 15 ſquares and 55 feet.

By Scale *and* Compaſſes.

In the firſt *Exnmple*, extend the compaſſes from 1 to 37.25, that extent will reach from 45 to 16 ſquares, and a little above three quarters of a ſquare.

In the ſecond *Example*, extend the compaſſes from 1 to 35.75, that extent will reach from 43.5 to 15 ſquares and 55 feet, that is, a little above a half ſquare.

2. *Of* Walling.

Bricklayers commonly meaſure their work by the rod-ſquare of 16 feet and a half; ſo that one rod in length and one in breadth, contain 272.25 ſquare feet; for 16.5 multiplied in itſelf, produces 272.25 ſquare feet: But in ſome places the cuſtom is to allow 18 feet to the rod, that is, 324 ſquare feet; and in ſome places the uſual way is to meaſure by the rood of 21 feet long and 3 feet high, that is, 63 ſquare feet; and here they never regard the thickneſs of the wall, but the uſual way is to moderate the price according to the thickneſs.

But in *Ireland* they meaſure by a perch of 21 feet in length, and 1 foot in height, without any regard to thickneſs in the meaſure, eſpecially in ſtone-work.

When you meaſure a piece of Brick-work, the firſt thing is to enquire by which of theſe ways it muſt be meaſured, then having multiplied the length and breadth in feet together, divide the product by the proper diviſor, either for rods, or roods, and the quotient is ſquare rods or ſquare roods accordingly.

But in *England*, commonly Brick walls, that are meaſured by the rod, are to be reduced to a Standard thickneſs, *viz.* of a brick and a half thick, (if it be not agreed on to the contrary) and to reduce a wall to ſtandard thickneſs, this is

The RULE.

Multiply the number of ſuperficial feet that are found to be contained in any wall, by the number of half bricks which that wall is in thickneſs, one third part of that product ſhall be the content thereof in feet, reduced to the ſtandard thickneſs of one brick and a half.

Example 1. If a wall be 72 feet, 6 inches long, and 19 feet 3 inches high, and 5 bricks and a half thick, how many rods of brick-work are contained therein when reduced to the ſtandard?

19.25 Height.
72.5 Length.

9625
3850
13475

1395.625
11

3)15351.875

272.25)5117.291(18 Rods.

239479

68.06)21679(3 Quarters of a Rod.

12.61

Anſwer. 18 Rods, 3 Quarters, 12 Feet.

F. I.
72 6
19 3

648
72
18 1 6
9 6 0

1395 7 6
11

3)15345

272)5115(18 Rods.

2395

68)219(3 Quarters of the Rod.

15

NOTE,

NOTE, that 68.06 is one fourth Part of 272.25.

NOTE alſo, That in reducing of Feet into Rods, they uſually reject the odd Parts and divide only by 272, as is done in that ſecond Way of the laſt *Example*; ſo the Anſwer, by that ſecond Way, is 18 Rods, 3 Quarters, and 15 Feet, more by about 2½ Feet than by the firſt Way, where it is done decimally; a Thing very inſignificant.

Example 2. If a Wall be 245 Feet 9 Inches long, and 16 Feet 6 Inches high, and 2 Bricks and a half thick; I demand how many Rods of Brick work are contained therein, when reduced to Standard Thickneſs?

```
           245.75
              16.5
          --------
           122875
          147450
         24575
         ---------
         4054.875
               5
       ----------
      3)20270
       ------
   272)6756(24 Rods.
        ------
         1316
        ------
     68)228(3 Quarter of a Rod.
        ------
           24
```

Anſwer. 24 Rods, 3 Quarters, 24 Feet.

F	I.		
245	9		
16	6		
1470			
245			
122	10	6	
12	0	0	
4054	10	6	*Anſwer* in Feet.

Before I ſhew how to work the two laſt *Examples* by ſcale and compaſſes, I will ſhew how to find proper diviſors to facilitate the operation, becauſe it would be too intricate and tedious to perform by ſcale and compaſſes, according to the rule above taught.

To find the proper Diviſors.

Divide 3 (the number of half bricks and 1 and ½) by the number of half bricks in the thickneſs, the quotient will be a diviſor which will give the anſwer in feet. But if you would have a diviſor to bring the anſwer into rods at once, then multiply 272.25 by the diviſor found for feet and the product will be a diviſor which will give the anſwer in rods.

Example. Let it be required to find a diviſor proper to reduce a wall to three bricks thick.

Divide 3 by 5, (the half bricks in the thickneſs) and the quotient is 5, which is a diviſor that will give the anſwer in feet. Then multiply 272.25 by .5, and the product is 136.125, the diviſor which will give the anſwer in rods; that is, as 136.125 is to the length of the wall, ſo is the height to the content in rods. Or, as .5 is to the length, ſo is the height to the content in feet.

After the ſame manner you may find diviſors for any other thickneſs, which you will find to be as expreſſed in the following little Table.

The Thickneſs of the Wall.	Diviſors for the Anſwer in Feet.	Diviſors for bringing the Anſwer in Rods.
1 Brick thick	1.5	408.375
1 and half Brick thick	1.	272.25
2 Bricks thick	.75	204.1875
2 and half Bricks thick	.6	163.35
3 Bricks thick	.5	136.125
3 and half Bricks thick	.4285	116.658
4 Bricks thick	.375	102.0937

Let the ſecond *Example*, aforegoing, be wrought by Scale and Compaſſes, where the length is 245.75, the height 16.5, and the thickneſs is 2 and half bricks.

Extend the compaſſes from 163.35 (the tabular number againſt 2 and half bricks) to 245.75, that extent will reach from 16.5 to 24 rods and 8 tenths.

Again if the length be 75 feet 6 inches, and the height 18 feet 9 inches, at 3 and half bricks thick, how many rods are contained therein ?

Extend the compaſſes from 116.678 (the tabular number) to 18.75, that extent will reach from 75.5 to 12.13 ; that is, 12 rod, and a little above half a quarter.

In *Britain* it will be very proper and commodious, for ſuch as have frequent occaſion to meaſure Brick-work, to have in the Line of Numbers little Braſs Center-pins at each of the Numbers in the third column of the above little Table, with a figure to denote the thickneſs of the Wall.

If a Wall be 104 feet 9 inches long, and 17 feet 3 inches high, how many rods are contained therein?

	F.	I.	
104.75	104	9	
17.25	17	3	
52375	728		
20950	104		
73325	126	2	3
10475	12	9	0
63) 1806.9375 (28	1806	11	3
126			
546			
504			
42			

Anſwer. 28 Rods, 42 Feet.

NOTE, That ſuch as dig Cellars do many times do them by the Floor, 18 feet ſquare, and a foot deep, being a Floor of Earth, that is, 324 ſolid feet.

3. *Of* Chimneys.

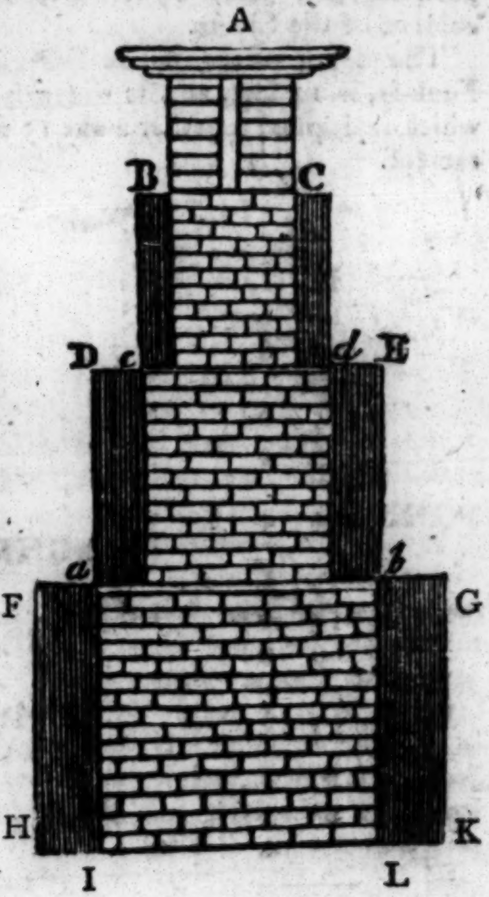
If you are to meaſure a Chimney ſtanding alone by itſelf, without any Party-wall being adjoined, then girt it about for the length and the height of the Story is the breadth; the thickneſs muſt be the ſame as the Jaums are of, provided that the Chimney be wrought upright from the Mantle-tree to the Cieling, not deducting any thing for the vacancy between the Floor (or Hearth) and the Mantle-tree becauſe of the gatherings of the Breaſt and Wings, to make room for the Hearth in the next Story.

If the Chimney-back be a Party-wall, and the Wall be meaſured by itſelf, then you muſt meaſure the depth of the two Jaums, and the length of the Breaſt for a length, and the height of the Story the breadth, at the ſame thickneſs your Jaums were of.

When you meaſure Chimney-ſhafts, girt them with a line round about the leaſt place of them, for the length, and the height ſhall be your breadth: and if they be four-inch Work, then you muſt ſet down your thickneſs at one Brick-work; but if they be wrought 9 inches thick (as ſometimes they are, when they ſtand high and alone above the Roof) then you muſt account your thickneſs one and half Brick, in conſideration of Wyths and Pargetting, and trouble in Scaffolding.

It is cuſtomary in moſt places to allow double meaſure for Chimneys.

Example. Suppoſe this Figure, ABCDEFGHK, to be a Chimney that hath a double Funnel towards the top, and a double Shaſt and is to be meaſured according to double meaſure.



Firſt, I begin with the Breaſt Wall IL, and the two angles LK and HI, which together are 18 feet 9 inches; then take the height of the Square HF, 12 feet 6 inches, which multiplied together produce 234 feet, 4 iuches, 6 parts, for the contents of the Figure FGHK.

For the Square D*a*E*b*, the length of the Breaſt Wall and two angles, is 14 feet 6 inches, and the height D*a* 9 feet, which multiplied together make 130 feet 6 inches, for the content of the Square D*a*E*b*.

Then the height of the next Square 7 feet, and the length of the Breaſt Wall and two angles is 10 feet 3 inches, which multiplied together produce 71 feet 9 inches, for the content of the Square B*c*C*d*.

The compaſs of the Chimney-ſhafts is 13 feet 9 inches, and the height 6 feet 6 inches, which multiplied together make 89 feet 4 inches 6 parts, the content of the Shafts.

The depth of the middle Fetter, that parts the Funnels, is 12 feet, and its wideneſs 1 foot 3 inches, which multiplied together make 15 feet, the content thereof.

See the WORK.

	F.	I.			
	18	9			18.75
	12	6			12.5
	225	0			9375
	9	4	6		3750
					1875
FGHK	234	4	6	FGHK	234.375
	F.	I.			
	14	6			14.5
	9	0			9
D*a*E*b*	130	6		D*a*E*b*	130.5
	F.	I.			
	10	3			10.25
	7	0			7
B*c*C*d*	71	9		B*c*C*d*	71.75
	F.	I.			
	13	9			13.75
	6	6			6.5
	82	6			6875
	6	10	6		8250
The Shaft	89	4	6	The Shaft	89.375

F. I.

	F.	I.
	1	3
	12	0
The fetter	15	0

1.25
12
The fetter 15.00

272) 1082 (3 rods.

68) 266 (3 quarters.

Rem. 62 feet.

	F.	I.	P.
FGHK	234	4	6
D*a*EB	130	6	0
B*c*C*d*	71	9	0
The ſhaft	89	4	6
The fetter	15	0	0
The ſum	541	0	0
The double	1082	0	0

Having added the five products together, and doubled the ſum, that double ſum is the content of the chimney in feet, according to double or cuſtomary meaſure; which feet muſt be reduced to rods, as was ſhewn before.

So the feet in the foregoing *Example* being reduced to rods (the thickneſs being ſuppoſed 1 and half bricks) it makes 3 rods, 3 quarters, and 62 feet, that is, 4 rods wanting 6 feet.

This is all the meaſure that can be allowed, when the chimney ſtands in a gable, or ſide-wall; in which caſe the back of the chimney (here not meaſured) is accounted as part of the gable; but if the chimneys ſtand by themſelves, as all ſtacks of chimneys in great buildings do, which, in ſuch caſe, is all chimney-work, and therefore ought to be meaſured double on all ſides.

§ III. *Of* PLAISTERERS Work.

THE Plaiſterers works are principally of two kinds, namely, 1. Works lathed or plaiſtered, which they call ceiling. 2. Works rendered, which

The compaſs of the Chimney-ſhafts is 13 feet 9 inches, and the height 6 feet 6 inches, which multiplied together make 89 feet 4 inches 6 parts, the content of the Shafts.

The depth of the middle Fetter, that parts the Funnels, is 12 feet, and its wideneſs 1 foot 3 inches, which multiplied together make 15 feet, the content thereof.

See the WORK.

	F.	I.			
	18	9			18.75
	12	6			12.5
	225	0			9375
	9	4	6		3750
					1875
FGHK	234	4	6	FGHK	234.375
	F.	I.			
	14	6			14.5
	9	0			9
DaEb	130	6		DaEb	130.5
	F.	I.			
	10	3			10.25
	7	0			7
BcCd	71	9		BcCd	71.75
	F.	I.			
	13	9			13.75
	6	6			6.5
	82	6			6875
	6	10	6		8250
The Shaft	89	4	6	The Shaft	89.375

F. I.

F.	I.
1	3
12	0
The fetter 15	0

1.25
12

The fetter 15.00

272) 1082 (3 rods.

68) 266 (3 quarters.

Rem. 62 feet.

	F.	I.	P.
FGHK	234	4	6
D*a*EB	130	6	0
B*c*C*d*	71	9	0
The ſhaft	89	4	6
The fetter	15	0	0
The ſum	541	0	0
The double	1082	0	0

Having added the five products together, and doubled the ſum, that double ſum is the content of the chimney in feet, according to double or cuſtomary meaſure; which feet muſt be reduced to rods, as was ſhewn before.

So the feet in the foregoing *Example* being reduced to rods (the thickneſs being ſuppoſed 1 and half bricks) it makes 3 rods, 3 quarters, and 62 feet, that is, 4 rods wanting 6 feet.

This is all the meaſure that can be allowed, when the chimney ſtands in a gable, or ſide-wall; in which caſe the back of the chimney (here not meaſured) is accounted as part of the gable; but if the chimneys ſtand by themſelves, as all ſtacks of chimneys in great buildings do, which, in ſuch caſe, is all chimney-work, and therefore ought to be meaſured double on all ſides.

§ III. *Of* PLAISTERERS Work.

THE Plaiſterers works are principally of two kinds, namely, 1. Works lathed or plaiſtered, which they call ceiling. 2. Works rendered, which is of two kinds, *viz.* upon brick walls, or between quarters in the partitions between rooms; all which are meaſured by the yard ſquare, or ſquare of 3 feet, which is 9 feet.

1. *Of* CEILING.

If a ceiling be 59 feet 9 inches long, and 24 feet 6 inches broad, how many yards doth that ceiling contain?

Multiply 59 feet 9 inches by 24 feet 6 inches, and the product is 1463 feet 10 inches, 6 parts; which divided by 9, the quotient is 162 yards, 5 feet.

F.	I.		
59	9		59 75
24	6		24.5
236			29875
118			23900
29	10	6	11950
18	0	0	9) 1463.875
1463	10	6	*Anſwer* 162.65

By Scale *and* Compaſſes.

Extend the compaſſes from 9 to 59 feet 9 inches, that extent will reach from 24 feet 6 inches to 162.5 yards.

2. *Of* Rendering.

Example. If the partitions between rooms be 141 feet 6 inches about, and 11 feet 3 inches high, how many yards are in thoſe partitions?

Multiply 141 feet 6 inches by 11 feet 3 inches, and the product is 1591 feet, 10 inches, 6 parts; which divided by 9, gives 176 yards 7 feet, the anſwer.

F.	I.		
141	6		141.5
11	3		11.25
1556	6		7075
35	4	6	2830
			1415
9) 1591	10	6	1415
F.	Y.		9) 1591.875
Anſw. 176	7		
			176.87

Anſwer, 176.87 Yards.

Extend the compaſſes from 9 to 141.5 that extent will reach from 11.25 to 176.87 yards.

NOTE 1. If there be any doors, windows, or the like, in your partitioning, you muſt make deductions for them.

NOTE 2. When you meaſure rendering upon brick walls, you are to make no deductions; but when you meaſure rendering between quarters, you may very well deduct one fifth part, for the quarters, braces, and interſtices.

NOTE 3. That whiting and colouring are both meaſured by the yard, as ceiling and rendering were; and as in rendering between quarters you deduct one fifth part, ſo in whiting and colouring you muſt add one fourth or one fifth part, at leaſt.

§ IV. *Of* JOINERS Work.

JOINERS meaſure their work by the yard ſquare; but in taking their dimenſions they differ from ſome others; for they have a cuſtom, and ſay, *we ought to meaſure where our plane touches*; wherefore,

fore, in taking the height of any room, where there is a cornice about, and ſwelling pannels and mouldings, they, with a ſtring, begin at the top, and girt over all the mouldings, which will make the room to meaſure much higher than it is: then, for meaſuring about the room, they only take it as it is upon the floor.

Example 1. If a room of wainſcot (being girt downwards over the mouldings) be 15 feet 9 inches high, and 126 feet 3 inches in compaſs, how many yards doth that room contain?

Multiply the compaſs by the height, and the product is 1988 feet, 5 inches, 3 parts; which divided by 9, gives 220 yards and 8 feet, the anſwer.

F.	I.	
126	3	
15	9	
630		
126		
63	1	6
31	6	9
3	9	0
9) 1988	5	3
Anſw. 220	8	

126.25	
15.75	
63125	
88375	
63125	
12625	
9) 1988.4375	
220	8

Facit 220 yards, 8 feet.

Example 2. If a room of wainſcot be 16 feet, 3 inches high (being girt over the mouldings) and the compaſs of the room 137 feet 6 inches, how many yards are contained therein?

Multiply 137 feet 6 inches by 16 feet 3 inches, and the product is 2234 feet, 4 inches, 6 parts; which divided by 9, the quotient is 248 yards and 2 feet.

F.	I.	
137	6	
16	3	
830		
137		
34	4	6
9) 2234	4	6
248	2	0

137.5	
16.25	
6875	
2750	
8250	
1375	
9) 2234.375	
248	2

Facit 248 yards, 2 Feet.

By Scale *and* Compaſſes.

For the firſt *Example*, extend the compaſſes from 9 to 126.25, that extent will reach from 15.75 to 220.8 yards.

For the ſecond *Example*, extend the compaſſes from 9 to 137.5, that extent will reach from 16.25 to 248 yards and about a quarter.

In Joiners work there is another thing to be obſerved, that is, in meaſuring of doors, window-ſhutters, and all ſuch work as is wrought on both ſides, they are paid for work and half work; ſo that in meaſuring all ſuch work, you muſt firſt find the content, as before, and take half that content and add to it; ſo ſhall the ſum be the content at work and half.

Example. If the window ſhutters about a room be 69 feet 9 inches broad, and 6 feet 3 inches high, how many yards are contained therein at work and half?

Multiply 69 feet 9 inches by 6 feet 3 inches, and the product is 435 feet, 11 inches, 3 parts; the half whereof is 217 feet, 11 inches, 7 parts; which added together, the ſum is 653 feet, 10 inches, 10 parts: which divided by 9, the quotient is 72 yards, 5 feet, the content at work and half.

F.	I.		
69	9		69.75
6	2		6.25
			34875
418	6		13950
17	5	3	41850
435	11	3	435.9375
217	11	7	217.9687
9) 653	10	10	653.9062
72	5		*Facit* 72 yards 5 feet.

By Scale *and* Compaſſes.

Extend the compaſſes from 9 to 69.75, that extent will reach from 6.25 to 48.4 yards; the half whereof is 24.2, which added together make 72.6 yards, the content at work and half.

NOTE, That you muſt make deductions for all window-lights; but you muſt meaſure the window-boards, ſopheta-boards and cheeks by themſelves.

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§ V. *Of* PAINTERS Work.

THE taking the dimenſions of Painters work is the ſame as that of Joiners, by girting over the mouldings and ſwelling pannels, in taking the height; and it is but reaſon that they ſhould be paid for that on which their time and colour are both expended. The dimenſions thus taken, the caſting up, and reducing feet into yards, is altogether the ſame as the Joiners work; but the Painter never requires work and half, but reckons his work, once, twice, or thrice coloured over. Only take notice, that window-lights, window-bars, caſements, and ſuch like things, they do at ſo much *per* piece.

Example. If a room be painted, whoſe height (being girt over the mouldings) is 16 feet 8 inches, and the compaſs of the room 97 feet 9 inches; how many yards are in that room?

Multiply 97 feet 9 inches by 16 feet 6 inches; and the product is 1612 feet 10 inches, 6 parts; which being divided by 9, the quotient is 179 yards, and 1 foot.

F.	I.		
97	9		97.75
16	6		16.5
584			48875
98			58650
48	10	6	9775
9)1612	10	6	9)1612.875(179.1
179	1		*Facit* 179 Yards, 1 foot.

By Scale *and* Compaſſes.

Extend the compaſſes from 9 to 16.5, that extent will reach from 97.75 to 179.2 yards.

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§ VI. *Of* GLAZIERS Work.

GLAZIERS meaſure their work by the foot ſquare, ſo that the length and breadth of a pane of glaſs in feet, being multiplied into each other, produceth the content.

NOTE, That Glaziers do uſually take their dimenſions to a quarter of an inch; and in multiplying feet, inches, and parts, the inch is divided into 12 parts, as the foot is, and each part ſubdivided into 12, *&c.*

Example 1. If a pane of glaſs be 4 feet, 8 inches and 3 quarters long, and 1 foot 4 inches, 1 quarter broad, how many feet of glaſs in that pane?

The decimal of $\left\{\begin{array}{l}8 \text{ Inches } \frac{3}{4}\\ 4 \text{ Inches } \frac{1}{4}\end{array}\right\}$ is $\left\{\begin{array}{l}\cdot 729\\ \cdot 354\end{array}\right.$

F.	I.	P.			
4	8	9			4.729
1	4	3			1.354
4	8	9			18916
2	6	11	0		23645
	1	2	2	3	14187
					4729
6	4	10	2	3	
					6.403066

Anſwer, 6 feet, 4 inches.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 1.354, that extent will reach from 4.729 to 6.4 feet, the content.

Example 2. If there be eight panes of glaſs, each 4 feet 7 inches, 3 quarters long, and 1 foot, 5 inches, 1 quarter broad; how many feet of glaſs is contained in the ſaid eight panes?

The decimal of $\left\{\begin{array}{l}7 \text{ Inches } \frac{3}{4}\\ 5 \text{ Inches } \frac{1}{4}\end{array}\right\}$ is $\left\{\begin{array}{l}.646\\ \cdot 437\end{array}\right.$

F. I.

F	I.	P.			4.646
4	7	9			1.437
1	5	3			
					32522
4	7	9			13938
1	11	2	9		18584
	1	1	11	3	4646
6	8	1	8	3	6.676302
				8	8
53	5	1	6	0	53.410416

Facit 53 feet, 5 inches.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 1.437, that extent will reach from 4.646 to 6.676: then extend the compaſſes from 1 to 8, that extent will reach from 6.676 to 53.4 the content.

Example 3. If there be 16 panes of glaſs, each 4 feet, 5 inches and a half long, and 1 foot, 4 inches 3 quarters broad, how many feet of glaſs are contained therein?

F.	I.	P.			4.458
4	5	6			1.395
1	4	9			
					22290
4	5	6			40122
1	5	10	0		13374
	3	4	1	6	4458
6	2	8	1	6	6.218910
				4	4
24	10	8	6	0	24.87564
				4	4
99	6	10	0	0	99.50256

Facit 99 feet, 6 inches.

NOTE, that inſtead of multiplying by 16, I have multiplied by 4 twice, becauſe 4 times 4 is 16.

By Scale *and* Campaſſes.

Extend the compaſſes from 1 to 1.395, that extent will reach from 4.4458 to 6.219; then extend the compaſſes from 1 to 16, that extent will reach from 6.219 to 99.5 feet, the content.

NOTE, That when windows have half rounds at the top, they meaſure them at the full height, as if they were ſquare. Alſo round or oval windows are meaſured at the full length and breadth of their diameters. Likewiſe crocket windows in ſtone-work are all meaſured by their full ſquares; and there is reaſon for ſo doing; for the trouble in taking the dimenſions to work by, the waſte of glaſs in working, and the time expended in ſetting up, is far more than the glaſs can be valued at.

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§ VII. MASONS Work.

MASONS meaſure their work ſometimes by the foot ſolid, ſometimes by the foot ſuperficial: and in ſome places they meaſure their walling by the rood, that is 21 feet long and 3 feet high, which is 63 feet.

Examples of each are as follow.

Examples 1. If a wall be 97 feet 5 inches long, 18 feet 3 inches high and 2 feet 3 inches thick, how many ſolid feet are contained in that wall?

F. I.

F.	I.			
97	5			97.417
18	3			18.25
776				487085
97				194834
24	4	3		779336
6	0	0		97417
1	6	0		
				1777.86225
1777	10	3		2.25
2	3			
				888930125
3555	8	6		355572050
444	5	6	9	355572050
4000	2	0	9	4000.1855625

Multiply the length, height and thickneſs together, and the laſt product is 4000 feet, 2 inches, the ſolid feet contained in the wall.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 18.25, that extent will reach from 97.417 to 1777.86; then extend from 1 to 1777.86, that extent will reach from 2.25 to 4000.18, the ſolid content.

Example 2. If a wall be 107 feet 9 inches long, and 20 feet 6 inches high, how many feet ſuperficial are contained therein?

F.	I.		
107	9		107.75
20	6		20.5
2155	0		53875
53	10	6	215500
2208	10	6	2208.875

Facit 2208 feet, 10 inches.

By Scale *and* Compaſſes.

Exend the compaſſes from 1 to 107.75, that extent will reach from 20.5 to 2208.875, the ſuperficial feet.

Example 3. If a wall be 112 feet 3 inches long, and 16 feet 6 inches high, how many roods are contained therein?

F.	I.	
112	3	
16	6	
676	0	
112		
56	1	6
1852	1	6

112.25
16.5
56125
67350
11225
63)1852.125(29
592
25

Facit 29 roods, 25 feet.

By Scale *and* Compaſſes.

Extend the compaſſes from 63 to 16.5, that extent will reach from 112.25 to 29.4 roods, the content.

CHAP.

CHAP. IV.

The Meaſuring of BOARD *and* TIMBER.

§ I. *Of* BOARD MEASURE.

TO meaſure a board, is no other but to meaſure a long ſquare.

Example 1. If a board be 16 inches broad, and 13 feet long, how many feet are contained therein?

Multiply 16 by 13, and the product is 208; which divided by 12, gives 17 feet, and 4 remains, which is a third part of a foot.

Or thus: Multiply 156 (the length in inches) by 16, and the product is 2496; which divided by 144, the quotient is 17 feet, and 48 remains, which is the third part of 144; the ſame as before.

As 12 : 13 : : 16
13

48
16

12)208

$17\frac{4}{12}$

Or, As 144 : 156 : : 16
16

936
156

144)2496($17\frac{48}{144}$

1056

48

By Scale *and* Compaſſes.

Extend the compaſſes from 12 to 13, that extent will reach from 16 to $17\frac{1}{3}$ feet, the content.

Or, extend from 144 to 156 (the length in inches) that extent will reach from 16 to $17\frac{1}{3}$ feet, the content.

Example 2. If a board be 19 inches broad, how many inches in length will make a foot?

Divide 144 by 19, and the quotient is 7.58 very near; and ſo many inches in length, if a board be 19 inches broad, will make a foot.

Inch. Inch. Inch. Inch.
As 19 : 144 : : 1 : 758 *fere*.

Extend the compaſſes from 9 to 144, that extent will reach from 1 to 7.58, that is, 7 inches and ſomething

thing more than a half. So if a board be 19 inches broad, if you take 7 inches and a little more than a half with your compaſſes from a ſcale of inches, and run that extent along the board, from end to end, you may find how many feet that board contains, or you may cut off from that board any number of feet deſired.

For this purpoſe there is a line upon moſt ordinary joint-rules, with a little table placed upon the end, of all ſuch numbers as exceed the length of the rule, as in this little table annexed.

I.	0	0	0	0	5	0	8½	6	Long.
F.	12	6	4	3	2	2	1	1	Long.
I.	1	2	3	4	5	6	7	8	Broad.

Here you ſee, if the breadth be one inch, the length muſt be 12 feet; if 2 inches, the length is 6 feet; if five inches broad, the length is about 2 feet, 5 inches, &c.

The reſt of the lengths are expreſſed in the line, thus: if the breadth be 9 inches you will find it againſt 16 inches, counted from the other end of the rule: if the breadth be 11 inches, then a little above 13 inches will be the length of a foot, &c.

§ II. *Of* SQUARED TIMBER.

BY ſquared timber is here meant all ſuch as have baſes, and the ſides ſtraight and parallel. The rules for meaſuring all ſuch ſolids are ſhewn in ſect. II. of Chap. 2. to which I refer you.

Example 1. If a piece of timber be 1 foot 3 inches (or 15 inches) ſquare, and 18 feet long, how many ſolid feet are contained therein?

```
      15
      15
    ----
      75
     15
    ----
     225
      18
    ----
    1800
    225
    ----
144) 4050 (28.125
    ----
    1170
     180
      360
       720
```

```
 F.  I.
  1  3
  1  3
 ------
  1  3
     3   9
 ------ ---
    1  6  9
          6
 ----------
    9  4  6
          3
 ------ ---
   28  1  6
```

Anſwer. 28 feet and half a quarter.

Here, inſtead of multiplying by 18 (where I wrought by feet and inches) I muſt multiply by 6, and then by 3, becauſe 3 times 6 is 18.

Example 2. If a piece of ſquared timber be 2 feet 9 inches deep, and 1 foot 7 inches broad, and 16 feet 9 inches long, how many feet of timber are in that piece?

Multiply the depth, breadth and length together, and the product will be the content.

```
       33
       19
     ----            F.  I.
      297             2   9
       33             1   7
     ----            --------
      627             2   9
    16.75             1   7   3
    -----            ----------
     3135             4   4   3
    4389             16   9
   3762              -------------
   627               69   8   0
   ------             3   3   2   3
144)10502.25(72.93   ----------------
   ------            72  11   2   3
   422
   1342
    465
   ----
    33
```

Anſwer, 72 feet, 11 inches, or 72 feet, 93 parts.

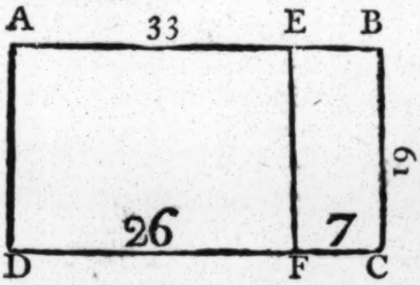
By Scale *and* Compaſſes.

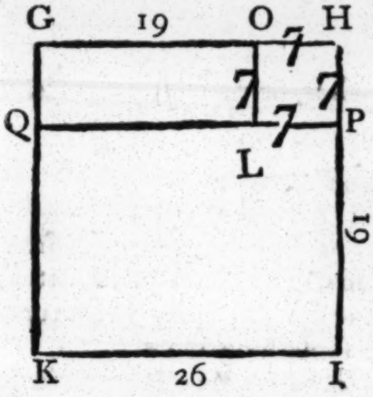
For the firſt *Example*, extend the compaſſes from 12 to 15 inches, (the ſide of the ſquare) that extent will reach from 18 feet the length, being twice turned over, to 28 feet, and ſomething more.

For the ſecond *Example*, find a mean proportional between 19 inches and 33 inches, by dividing the ſpace between them into two equal parts, and the compaſs point will reſt upon 25, which is a mean proportional between 19 and 33.

Then extend the compaſſes from 12 to 25, (the proportional found) that extent will reach (being twice turned over) from 16.75 feet, the length, to 72.93 feet, the content.

A common error is committed, for want of art, in meaſuring theſe laſt ſorts of ſolids, by adding the depth and breadth together, and taking half for the ſide of a mean ſquare. This error, though it be but ſmall when the depth and breadth are pretty near equal, yet if the difference be great the error is very conſiderable; for the piece of timber, thus meaſured, will be more than the truth, by a piece, whoſe length is equal to the length of the piece of timber to be meaſured, and the ſquare equal to half the difference of the breadth and depth, as I ſhall here demonſtrate.





I ſay, the ſquare GHIK is greater than the parallelogram ABCD, by the little ſquare OHPL; for the parallelogram QPIK is equal to the parallelogram AEFD, and the parallelogram GOLQ is equal to the paralellogram EBCF. Therefore the ſquare is greater than the paralellogram by the little ſquare OHPL. —Which was to be proved.

Otherwiſe, you may prove it by numbers, thus: the ſum of 33 and 19 is 52; the half thereof is 26; the ſquare of 20 is 676; and the product of the depth and breadth is 627; the difference of theſe two is 49, equal to the ſquare of half the difference; for the difference between 33 and 19 is 14, the half thereof is 7, whoſe ſquare is 49. Which was to be proved.

Now, if this be multiplied by the length of the piece, and that product divided by 144, to bring it to feet, and thoſe feet added to the true content, the ſum will be equal to the content found by the falſe way mentioned.

See the work of both.

33 Depth. 16.75 the length.
19 Breadth. 49 the Squ. of half diff.

52 Sum. 15075
6700

26 Half.
26 144) 820.75 (5.69

156 1007
52 1435

676 139
16.75

3380
4732
4056
676

144) 11323.00 (78.63

1243
910
460

28

Feet.
To 72.93 the true content.
Add 5.69 the part ſuperfluous.

Sum 78.62 equal to the content by the falſe way.

By Feet *and* Inches.

F.	I.				F.	I.	
0	7				2	2	
0	7				2	2	
0	4	1			4	4	
16	9					4	4
5	5	4			4	8	4
	3	0	9		16	9	
5	8	4	9	Part ſuperfluous.	75	1	4
72	11	2	3	true content add.	3	6	3
78	7	7	0	=con. the falſe way.—fal. con.	78	7	7

To find how much in length makes a foot of any ſquared timber.

Always divide 1728 (the ſolid inches in a foot) by the area of the baſe, the quotient is the length of a foot.

This rule is general for all timber, which is of equal thickneſs from end to end, whether it be ſquare, triangular, multangular, or round.

Example 1. If a piece of timber be 18 inches ſquare, how much in length will make a foot ſolid?

```
     18
     18
    ----
    144
     18
    ----
    324)1728($5\frac{1}{3}$
        ----
        1620
         108
```

Anſw. 5 inches and 1 third.

By

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 18, that extent will reach from 18 to 324, the ſquare or area of the baſe; then extend from 324 to 1728, that extent will reach down from 1 to 5 inches and $\frac{1}{3}$ of an inch.

Or thus: extend the compaſſes from 18 to 41.569, that extent (turned twice over from 1) will at laſt fall upon $5\frac{1}{3}$, as before.

NOTE, That 41.569 is the ſquare root of 1728.

Example 2. If a piece of timber be 22 inches deep, and 15 inches broad, how much in length will make a foot?

22
15
———
110
22
———
330)1728(5.23
———
780
1200
———
210

Anſwer. 5 inches and 23 parts.

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 15, that extent will reach from 22 to 330; then extend from 336 to 1728, that extent will reach from 1 to 5.23 inches, the length of a foot.

There is a line for this purpoſe upon moſt ordinary rules, with a little table at the end, of all ſuch numbers as exceed the length of the rule ſuch as this annexed.

0	0	0	0	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of ſq.

Here you ſee, if the ſide of the ſquare be 1, the length muſt be 144 feet; if two inches be the ſide of the ſquare, it muſt be 36 feet in length, to make a ſolid foot, &c.

If the ſide of the ſquare be not in the little table, you will find it upon the line, thus; if the ſide of the ſquare be 16 inches, you will find it againſt 6 inches and 7 tenths, counted from the other end of the rule.

Then, if you take the length of a foot from the line of inches with your compaſſes, and run the compaſſes along the piece from end to end, you will find how many feet are contained in that piece, or you may cut off any number of ſolid feet that ſhall be deſired; but if the ſides of the piece be unequal, find a mean proportional number, as is before taught, by dividing the diſtance upon the line of numbers into two equal parts, thus: if the breadth be 25 inches, and the depth 9 inches, divide the ſpace upon the line of numbers into two equal parts, and you will find the middle point at 15, ſo is 15 inches the geometrical mean proportional ſought; then, if you look for 15 upon the line above mentioned, that 7 inches, and a little above half, will be the length of a foot.

§ III. *Of unequal* Squared Timber.

BY unequal ſquared timber, I mean all ſuch as have unequal baſes, that is, ſuch as is thicker at one end than at the other; and ſuch are moſt timber trees, when they are hewn, and brought to their ſquares.

The uſual way to meaſure ſuch timber, is to take a ſquare about the middle of the piece, which they take to be a mean ſquare: This way, when the piece is pretty near as thick at one end as the other, is ſomething near the truth; but when there is a great diſproportion between the ends of the piece, the error is conſiderable. All ſuch ſolids being the fruſtums of pyramids, the true way of meaſuring them muſt be by Sect. VII. Chap. 2. I ſhall now give an *Example* or two which I will work both by the true and falſe ways, whereby you will ſee the difference.

Example 1. If a piece of timber be 25 inches ſquare at the greater end, and 9 inches ſquare at the leſſer end, and 20 feet long, how many feet of timber are in that tree?

```
      25
       9
     ———
Sum   34
     ———
Half  17  The ſide of the ſquare in the middle.
      17
     ———       By rule II. ſect. VII. chap. 2.
     119                        25
      17          25             9
     ———           9           ———
     289         ———            16 Diff. of the ſides.
      20         225            16
     ———                       ———
144)5780(40.13                  96
     ———                        16
     200                       ———
     560                     3)256 The ſquare.
     ———                     85.3333
     128                      225
                             ———————
                             310.3333
```

Anſwer. 40.13 feet by the falſe way.

310.3333
20

144)6206.6660(43.101

446
146
266

122

Anſwer. 43.101 feet by the true way; ſo that there is near three feet difference.

By Scale *and* Compaſſes.

Extend from 1 to 9, that extent will reach from 25 (the ſame way) to 225; the rectangle of the ſides of the two baſes; then the difference between the ſaid ſides is 16: extend from 3 to 16, that extent will reach from 16 to 85.333, a third part of the ſquare; which added to 225, the ſum is 310.333, a mean area: then extend from 144 to 310.333, that extent will reach from 20 (the length) to 43.1 foot (the content) the true way.

Extend the compaſſes from 12 to 17, the ſide of the middle ſquare, that extent will reach from 20, the length being twice turned over, to 40.1 feet, the content by the falſe way.

Example 2. If a piece of timber be 32 inches broad and 20 inches deep at the greater end, and 10 inches broad and 6 deep at the leſſer end, and 18 feet long, how many feet of timber are in that piece?

Rule I. Sect. VII. Chap. 2.

32	6
20	10
640	60
60	
....	
38400	38400

. . :.

38400 .... (195.959 Mean proportional

1 640. The greater baſe.

60. The leſſer baſe.

29)284

385)2300 895.959 The ſum.

3909)37500 $6 = \frac{1}{3}$ height.

39185)231900

391909)3597500 5375.754

144)5375.754(37.33

1055

477

455

23

Add. { 32 / 10 } 20 / 6 } Add.

Sum. 42 26 Sum.

Half. 21 13 Half.

13

63

21

273 Area in the middle.

18 Length.

2184

273

144)4914(34.12

594

180

360

72 Feet.

Anſw. { Content the true way 37.33
Content the falſe way 34.12 }

By Scale *and* Compaſſes.

Extend the compaſſes from 1 to 20, that extent will reach from 32 to 640, the area of the greater baſe.

Then extend from 1 to 10, that extent will reach from 6 to 60, the area of the leſſer baſe. Then extend from 1 to 60, that extent will reach from 640 to 38400, the product of the two areas: Find the ſquare root thereof, by dividing the ſpace between 1 and 38400 into two equal parts, ſo you will find the middle point at 195.959, the root ſought; which is a mean proportional between the greater and the leſſer areas. Then add the mean proportional and two areas together, and the ſum is 895.959; which multiplied by 6 (a third part of the length) by extending from 1 to 6, that extent will reach from 895.959 to 5375.75. Then extend from 144 to 5375.75, and that extent will reach from 1 to 37.33 feet, the true content.

For the falſe way, half the ſum of the breadths is 21, which is the breadth in the middle; and half the ſum of the depths is 13: Extend from 1 to 13, that extent will reach from 21 to 273, the area of the middle baſe. Then extend from 144 to 273, that extent will reach from 18, the length, to 34.12, the content the falſe way.

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§ IV. *Of* Round Timber, *whoſe Baſes are equal.*

THE uſual way to meaſure round timber trees is to girt them about the middle with a ſtring, and take the fourth part of that girth for the ſide of a ſquare, by which they meaſure the piece of timber as if it was ſquare.

But that this is an error, I ſhall make appear as follows: If the circumference of a circle be 1, the area will be .07958; then the fourth part of 1 is

.25,

.25, which ſquared makes .0625, this they take for a mean area inſtead of .07958: Therefore the true content always bears ſuch proportion to the content found by the aforeſaid cuſtomary falſe way, as .07958 to .0625; which is nearly as 23 to 18, ſo that in meaſuring by that cuſtomary falſe way, there is above the one fifth part of what the true content ought to be.

This error, though it has been ſo often confuted, yet it is grown ſo cuſtomary in all places, that there is little hopes of my prevailing with men that are ſo wedded to it, to embrace the truth; I ſhall therefore, in the following *Examples*, ſhew how to work both the true way, and alſo the falſe or cuſtomary way.

Example 1. If a piece of timber be 96 inches in circumference, or girth, and 18 feet long, how many feet of timber are contained therein?

A fourth part of 96 is 24
24
———
96
48
———
576 Area Baſe.
18
———
4608
576
———
144) 10368 (72
1008
———
288
288
———
...

Content the falſe way 72 feet.

Or thus.
F. I.
2 0
2 0
———
4 0
18
———
72 0

Then the true way.

```
          96
          96
        ----
         576
        864
        ----
        9216
      .07958
      ------
       73728
      46080
     82944
    64512
  ----------
   733.40928  The area by Prob. 5 Sect. IX.
          18                        [Ch. 2.
  ----------
   586727424
   73340928
  ----------
144) 13201.36704 (91.67
     -----
      241        The true content 91.67 feet.
       973
        1096
       -----
          88
```

By Scale *and* Campaſſes.

Extend from 12 to 24 (the fourth part of the girth) that extent turned twice over from 18 feet (the length) will at laſt fall upon 72 feet, the content the cuſtomary way.

Extend from 42.54 to 96 (the girth) that extent will reach from 18 feet turned twice over, to 91.67 feet, the true content.

Example 2. If a piece of timber be 86 inches girth, and 20 feet long, how many feet are contained therein?

The fourth part of 86 is 21.5

21.5

1075

215

430

462.25

20

144)9245.00(64.2

605

290

20

F.	I.	P.	
1	9	6	
1	9	6	
1	9	6	
1	4	1	6
	0	10	9
3	2	6	3
			20
64	2	5	0

The Content the falſe way 64.2 feet.

By the true way.

86

86

516

688

7396

```
       7396
     .07958
   --------
      59168
     36980
    66564
   51772
   --------
   588.57368
          20
   ---------
144) 11771.47360 (81.74
     -----------
     251
     1074
       667
     -----
       91
```

The true content 81.74 feet.

By Scale *and* Compaſſes.

Extend from 12 to 21.5, that extent turned twice over from 20, will reach at laſt to 64.2 feet, the content the falſe way.

Extend from 42.54 to 86, that extent turned over from 20, will at laſt fall upon 81.74 feet, the true content.

Theſe cylindrical proportions may be very eaſily wrought upon the line numbers.

Problem 1. Having the diameter of a cylinder in inches, to find the length of a foot.

Suppoſe the diameter is 22.6 inches.

As 22.6 : to 46.9 : : ſo is 1 to a fourth number, and that to the length of a foot in inches, 4.3

Extend the compaſſes from 22.6 to 46.9, that extent will reach from 1 to a fourth number, then turn them over again, and that will reach to 4.3 inches.

Problem 2. Having the diameter in foot-meaſure, to find the length of a foot in foot-meaſure.

Suppoſe the diameter 1.88 feet.

Then, as 1.88 : to 1.128 : : ſo is 1 : to a fourth number; and ſo is that to the length of a foot in foot-meaſure .358.

Extend the compaſſes from 1.88 to 1.128, that extent turned twice from 1, will reach to 358 parts of a foot.

Problem 3. Having the circumference in inches, to find the length of a foot in inches.

Suppoſe the circumference 71 inches.

Then, as 71 : to 147.36 : : ſo is 1 to a fourth number, and ſo is that to the length of a foot in inches 4.3.

Extend the compaſſes from 71 to 147.36, that extent turned twice from 1, will reach to 4.3 inches, the length of a foot.

Problem 4. Having the circumference in foot-meaſure, to find the length of a foot in foot-meaſure.

Suppoſe the circumference 5.92 feet.

Then, as 5.92 : to 3.545 : : ſo is 1 : to a fourth number; and ſo is that to the length of a foot in foot-meaſure .358.

Extend the compaſſes from 5.92 to 3.545, that extent turned twice over from 1, will fall upon .358 part of a foot.

Problem 5. Having the diameter in inches, and length in inches, to find the content in inches.

Suppoſe the diameter is 22.6 inches, and the length is 156 inches, or 13 feet.

Then, as 1.128; to 22.6 : : ſo is 156 : to a fourth number; and ſo is that to the content in inches 62674.

Extent the compaſſes from 1.128 to 22.6, that extent turned twice from 156, will fall upon 62674 inches the content.

NOTE, That 1.128 is the diameter when the ſide of the ſquare equal is = 1.

Problem 6. Having the diameter in foot-meaſure, and length in feet, to find the content in feet.

Suppoſe the diameter 1.88 feet, and the length 13 feet.

Then, as 1.128 : to 1.88 : : ſo is 13 to a fourth number; and ſo is that to the content in feet 36.27.

Extend from 1.128 to 1.88, that extent turned twice from 13, will fall upon 36.27.

Problem 7. Having the diameter in inches, and length in inches to find the content in feet.

Suppoſe the diameter 22.6 inches, and the length 156 inches.

Then, as 46.9 : is to 22.6 : : ſo is 156 : to a fourth number; and ſo is that to the content in feet 36.27.

Extend from 46.9, to 22.6, that extent turned twice from 156, will fall upon 36.27 feet, the content.

NOTE, That 46.9 is the diameter of a circle whoſe area is 1728.

Problem 8. Having the diameter in inches, and length in feet, to find the content in feet.

Suppoſe the diameter 22.6 inches, and the length 13 feet.

Then, as 13.54 : to 22.6 : : ſo is 13 to a fourth number; and ſo is that to the content in feet 36.27.

Extend from 13.54 to 22.6, that extent turned twice from 13, will fall upon 36.27.

NOTE, That 13.54 is the diameter of a circle when the area is 144.

Problem 9. Having the circumference in inches, and length in inches, to find the content in inches.

Suppoſe the circumference 71, and the length 156 inches.

Then, as 3.545 : is to 71 : : ſo is 156 to a fourth number; and ſo is that to 62674, the content in inches.

Extend the compaſſes from 3.545 to 71, that extent turned twice from 156, will fall upon 62674, the content.

NOTE, That 3.545 is the circumference when the ſide of the ſquare is equal 1.

Problem 10. Having the circumference in feet, and length in feet, to find the content in feet.

Suppoſe the circumference 5.92 feet, and length 13 feet.

Then, as 3.545 : to 5.92 : : ſo is 13 : to a fourth number; and ſo is that to 36.27.

Extend from 3.545 to 5.92, that extent turned twice from 13, will fall upon 36.27 feet, the content.

Problem 11. Having the circumference in inches, and length in inches, to find the content in feet.

Suppoſe the circumference 71 inches, and length 156 inches.

Then, as 147.36 : to 71 : : ſo is 156 to a fourth number; and ſo is that to the content in feet 36.27.

Extend the compaſſes from 147.36 to 71, that extent turned twice from 156, will fall upon 36 27 feet, the content.

NOTE, That 147.36 is the circumference of a circle whoſe area is 1728.

Problem 12. Having the circumference in inches, and length in feet, to find the content in feet.

Suppoſe the circumference 71 inches, and length 13 feet.

Then, as 42.54 : is to 71 : : ſo is 13 : to a fourth number ; and ſo is that to the content in feet 36.27.

Extend the compaſſes from 42.54 to 71, that extent turned twice from 13, will reach to 36.27 feet, the content.

NOTE, That 42.54 is the circumference of a circle whoſe area is 144.

§ V. *Of* Round Timber, *whoſe Baſes are unequal.*

THE uſual way to meaſure round timber, as I ſaid before, is to take a fourth part of the girth in the middle of the piece, for the ſide of a mean ſquare. But this way I have proved to be erroneous in timber that is all the way of an equal thickneſs ; and it muſt be much more ſo in timber that is tapering, and the more tapering it is the greater is the error : for to the error in the laſt ſection, there is added the error in the third ſection ; therefore, to meaſure all ſuch timber according to art and truth, ſuch a piece ought to be conſidered as a fruſtum of a cone, and ſhould be meaſured by the rules given in Sect. VIII. Chap. 2. by which rules the following *Examples* are wrought.

Example 1. If a piece of timber be 9 inches diameter at the leſſer end, and 36 inches at the other end, and 24 feet long, how many feet of timber are therein?

```
         36          36 } Subtract.
          9           9 }
        ----      ------
Rect.   324           27 Difference.
                      27
                  ------
                     189
                     54
                  ------
                  3)729 The Square.
                  ------
                     243 One third.
                     324 Rectangle add.
                  ------
                     567
                  ------
                    7854
                     567
                 -------
                   54978
                  47124
                 39270
                ---------
A mean area  445.3218
                     24
                ---------
               17812872
               8906436
                ---------
       144) 10687.7232 (74.22
                607
                  317
                   292
                 -----
                     4      Anſwer 74.22 Feet.
```

Or thus, by Feet and Inches.

F. I.
3 0
0 9
2 3 Rect.

F. I.
2 3 Difference.
2 3
4 6
0 6 9
5 0 9 the ſquare.
1 8 3 one third.
2 3 0 Rectangle added.
3 11 3 a mean ſquare.

F. I. P.
Then, as 14 : to 11 :: ſo is 3 11 3 to the area.
11
7) 43 3 9
2) 6 2 3
3 1 1 6
6
18 6 9 0
4
74 3 0 0

Here, inſtead of dividing by 14, I divide by 7 and by 2, becauſe twice 7 is 14.

And inſtead of multiplying by 24 feet, the length, I multiply by 6 and 4, becauſe 6 times 4 is 24.

By Scale and Compaſſes this is too troubleſome.

Example 2. If a piece of timber be 136 inches circumference at one end, and 32 inches circumference at the other end, and 21 feet long, how many feet of timber are contained in that piece?

```
 136      136
  32       32
 ----     ----
 272      104 Difference.
408       104
 ----     ----
4352      416
         1040
         -----
      3) 10816 the ſquare.
         ----------
           3605.333 one Third.
           4352     Rectangle add.
           --------
           7957.333 a mean circumf. ſquared.
             .07958
           --------
          63658664
         39786665
        71615997
       55701331
       -------------
       633.24456014 the mean area.
                 21
      --------------
       633.24456014
     126648912028
     ---------------
     13298.13576294

       144) 13298.13 (92.34
            --------
              338
               501
                693
            -------
                117
```

Anſwer 92.34 feet.

By Feet and Inches thus.

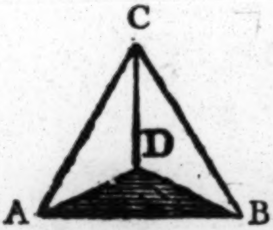
F.	I.			F.	I.			
11	4			8	8			Difference.
2	8			8	8			
22	8			69	4			
7	6	8		5	9	4		
30	2	8	3)	75	1	4		The ſquare.
				25	0	5	4	
				30	2	8	0	
				55	3	1	4	Squ. of the circumfer.

	F.	I.	P.	S.	
As 88 : to 7 : :	55	3	1	4	: to the mean area.
				7	
11)	386	9	9	4	
8)	35	1	11	9	
	4	4	8	11	The mean area,
				7	
	30	9	2	5	
				3	
Facit	92	3	7	3	

§ VI. *Of the five* Regular Bodies.

THESE bodies may be all meaſured by the fourth Section of Chap II. except it be the cube, or hexaedron, which is already meaſured in Sect. I. of that chapter.

1. *Of the* TETRAEDRON.



A Tetraedron is a ſolid contained under four equal and equilateral triangles.

Let ABCD be a tetraedron whoſe ſide is 12 inches, the perpendicular height, 9.798 inches.

By Sect. V. Chap. I. the area of the triangle will be found 62.352 ; a third part of it is 20.784, which multiplied by the perpendicular height, the product is 203.641632 ſolid inches, the content.

10.392 The perpendicular of the triangle.
6 Half the ſide.

62.352 Area of the triangle.

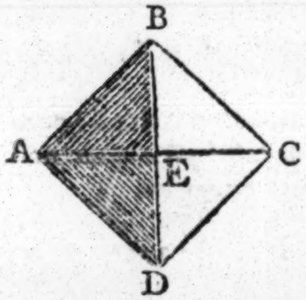
20.784 One third part.
9.798 The perpedicular height.

166272
187056
145488
187056

203.641632 The ſolidity.

The ſuperficial content is 4 times the area of the triangle, *viz.* 249.408 inches, becauſe there are 4 triangles.

2. *Of the* OCTAEDRON.



The Octaedron is a body contained under eight equal and equilateral triangles.

Let ABCDE be an octaedron whoſe ſide is 12 inches; the content ſolid and ſuperficial is required.

An Octaedron is compoſed of two quadrangular pyramids joined together by their baſes; therefore, if the area of the baſe be multiplied into a third part of the length of both pyramids, the product will be the ſolid content.

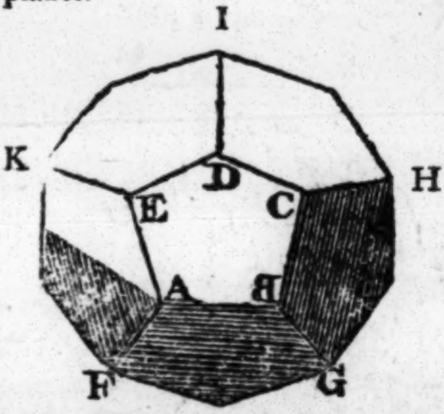
```
  5.6568 A third part of the length.
     144 Area of the ſquare baſe.
  ------
  226272
 226272
56568
--------
814.5792 The ſolidity.
```

The ſuperficial content will be juſt double to that of the tetraedron, *viz.* 498.816, becauſe the ſide of this is ſuppoſed to be equal to the ſide of that, and becauſe the octaedron is contained under eight triangles, and the tetraedron but under four.

3. *Of*

3. *Of the* DODECAEDRON.

The Dodecaedron is a ſolid body, contained under twelve pentangular planes.



Let ABCDEFGHIK be a dodecaedron, each ſide thereof being 12 inches; the content ſolid and ſuperficial is required.

The ſolidity of the dodecaedron is compoſed of 12 pentangled pyramids, whoſe vertices all meet in the center. Therefore if we find the ſolidity of one of thoſe pyramids, and multiply that by 12 that product will be the ſolidity of the dodecaedron.

The altitude of one of the pentangled pyramids will be found to be 13.36219.

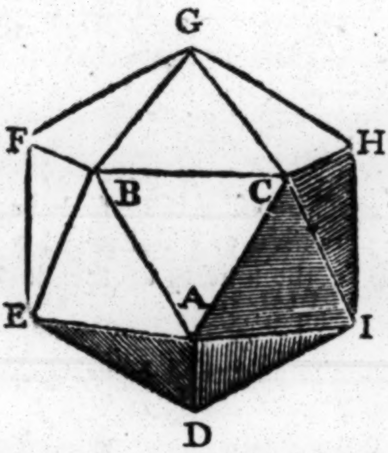
The perpendicular of the pentagon will be

8.258292	
30	Half ſum of the ſides.
247.748760	Area of the pentagon.
60454.4	A third part of 13.36219 inverted.
99099504	
9909950	
1238744	
99099	
1486	
1103.48783	Content of one pyramid.
12	
13241.85396	The ſolidity of the dodecaedron.

If the area of the pentagon be multiplied by 12, the product will be the ſuperficial content.

```
  247.74876
         12
-----------
2972.98512 The ſuperficial content.
```

4. *Of the* ICOSAEDRON.



The Icoſaedron is a ſolid body, contained under 20 equal and equilaterial triangles.

Let ABCDEFGHI be an Icoſaedron, each ſide thereof being 12 inches; the content ſolid and ſuperficial is required.

The icoſaedron is compoſed of twenty triangular pyramids, with their vertices all joining in the center.

Therefore, if the ſolid content of one pyramid be multiplied by 20, the product is the whole ſolid content of the icoſaedron.

10.39224 The perpendicular of the triangle.
6 Half the ſide.

62.35344
20

1247.06880

3.0230456 The third part of the altitude of the [pyramid.
44353.26

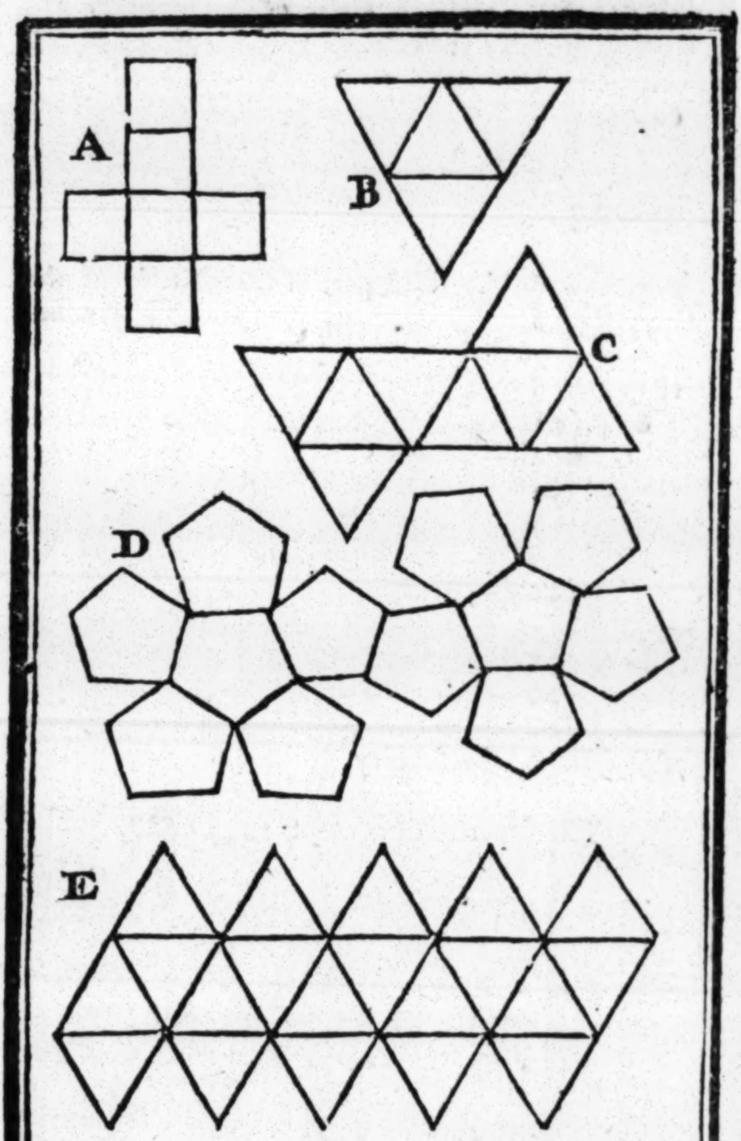
181382736
6046091
906914
151152
9069
1209
121

188.497292
20

3769.945840 The ſolidity.

The ſuperficial content 1247.0688.

A The



A The Cube. B Tetraedron. C Octaedron. D Dodecaedron. E Icoſaedron.

By theſe figures you may cut theſe bodies in fine paſteboard, cutting all the lines half through, and ſo turn them up and glue them.

A TABLE *ſhewing the ſolidity and ſuperficial content of any of the Regular Bodies, the ſide being* 1, *or unity.*

	The Names of the Bodies.	Solidity.	Superficies.
B	Tetraedron	0.1178511	1.732051
C	Octaedron	0.4714045	3.464102
A	Hexaedron	1.0000000	6.000000
E	Icoſaedron	2.181695	8.660254
D	Dodecaedron	7.663119	20.645729

By this table the content, either ſuperficial or ſolid, of any of theſe bodies may very readily be found; for all like ſuperficial figures are in proportion one to another, as are the ſquares of their like ſides: therefore it will be, as the ſquare of 1 (which is 1) : is to the ſuperficial content in the table : : ſo is the ſquare of the ſide of the like body : to the ſuperficial content of the ſame body. Therefore, if the number in the table be multiplied by the ſquare of the ſide given, the product is the ſuperficial content required.

Again. All like ſolids are in ſuch proportion to each other as are the cubes of their like ſides; therefore it will be, as 1 : (which is the cube of 1) is to the ſolid content in the table : : ſo is the cube of the ſide given : to the ſolid content required. Therefore, if the number in the table be multiplied by the cube of the given ſide, the product will be the ſolid content of the ſame body.

Example 1. If the ſide of a dodecaedron be 12 inches (as before) what is the content ſolid and ſuperficial?

7.663119 The tabular number.
1728 The cube of the ſide.

61304952
15326238
53641833
7663119

13241.869632 The ſolid content nearly the ſame (as before.

20.645729 The tabular number.
144 The ſquare of the ſide.

82582916
82582916
20645729

2972.984976 The ſuperficial content.

By Scale *and* Compaſſes.

Extend from 1 to 12 (the ſide) that extent being turned three times over from 7.663119, will at laſt fall upon 13241.86, *&c.* the ſolid content.

And if you apply the ſame extent twice from 20.645729, it will at laſt fall upon 2972.98, *&c.* the ſuperficial content.

Example 2. If the ſide of an octaedron be 20 inches, what is the content ſolid and ſuperficial?

.4714045 The tabular number.
8000 The cube of the ſide.

3771.2360000 The ſolid content.

3.464102 The tabular number.
400 The ſquare of the ſide.

1385.640800 The ſuperficial content.

By

By Scale *and* Compaſſes.

Extend from 1 to 20, that extent turned three times over from .4714045; will at laſt fall upon 3771.236, the ſolid content. The ſame extent turned twice over from 3.464, *&c.* will at laſt fall upon 1385.64, the ſuperficial content.

§ VII. *How to meaſure any irregular Solid.*

IF you have any piece of wood or ſtone that is craggy and uneven, and you deſire to find the ſolidity, put the ſolid into any regular veſſel, as a tub, a ciſtern, or the like, and pour in as much water as will juſt cover it; then take out the ſolid, and meaſure how much the fall of the water is, and ſo find the ſolidity of that part of the veſſel.

Example. Suppoſe a piece of wood or ſtone to be meaſured, and ſuppoſe a tub 32 inches diameter, into which let the ſtone or wood be put and covered with water; then when the ſolid is taken out, ſuppoſe the fall of the water 14 inches; ſquare 32, and multiply the ſquare by .7854, the product will be 804.2496, the area of the baſe; which multiplied by 14, the depth or fall of the water, and the product is 11259.49, *&c.* which divided by 1728, the quotient is 6.51 feet; and ſo much is the ſolid content required.

CHAP. V.

Practical Queſtions in MEASURING.

Queſtion 1. IF a pavement be 47 feet 9 inches long, and 18 feet 6 inches broad, I demand how many yards are contained therein?

F.	I.		
47	9		47·75
18	6		18.5
376	0		23875
47			38200
23	10	6	4775
9	0	0	
4	6	0	9) 883·375
9) 883	4	6	98.1

Anſwer. 98 yards 1 foot.

Queſt. 2. There is a room whoſe length is 21.5 feet, and the breadth 17.5 feet, which is to be paved with ſtones each 18 inches ſquare; I demand how many ſuch ſtones will pave it?

21.5 1.5
17.5 1.5
1075 75
1505 15
215 2.25 Area of one ſtone.
2.25)376.25(167
1512
1625
50 *Anſw.* 167 ſtones.

Queſt. 3. There is a room 109 feet 9 inches about, and 9 feet 3 inches high, which is all (except two windows, each 6 feet 6 inches high, and 5 feet 9 inches broad) to be hung with tapeſtry that is ell broad; I deſire to know how many yards will hang the ſaid room.

From the content of the room ſubtract the content of the windows, and divide the remainder by the ſquare feet in a yard of tapeſtry.

3.75 109.75 Length. 5.75
3 9.25 Breadth. 6.5
11.25 54875 2875
21950 3450
98775 37375
1015.1875 Content of the room. 2
74.75 Cont. of the windows ſub. 74.75
11.25) 940.4375 (83.59
4043
6687
10625
500 *Anſwer* 83.59 Yards.

Queſt. 4. If the axis of a globe be 27.5 inches, I demand the content ſolid and ſuperficial.

3.1416
27.5

157080
219912
62832

86.39400 The circumference.
27.5 The diameter.

431970
604758
172788

6) 2375.8350 The ſuperficial cont.

395.9725 A ſixth part.
27.5

19798625
27718075
7919450

10889.24375 The ſolidity in inches

Anſwer { 6.3 Feet ſolid.
16.49 Feet ſuperficial.

Queſt. 5. There is the fruſtum of a globe, the diameter of whoſe baſe is 24 inches, and the altitude thereof is 10 inches; what is the content ſolid and ſuperficial?

Find the ſuperficies, as is directed in *Page* 177, and find the ſolidity by the firſt theorem in *Page* 188.

24
24

96
48

576

.7854
576

47124
54978
39270

452.3904 } add.
314.16 }

.7854
400

314.16000

20
20

400

766.

766.5504 The curve ſuperficies.
452.3904 The baſe add.

1218.9408 The whole ſuperficial content in inches.
12 × 12 = 144
3

432
100 The ſquare of the altitude add.

532 The Sum.
10 Multiply by the altitude.

5320
.5236 Multiply

31920
15960
10640
26600

2785.5520 The ſolidity in inches.

Queſt. 6. If a tree girt 18 feet 6 inches, and be 24 feet long, how many tuns of timber are contained in that tree?

F. I.
4)18 6 The Girt.

4 7 6 the 4th part.
4 7 6

18 6 0
2 8 4 6
2 3 9

21 4 8 3 Here I multiply by 6 and by
6 4, becauſe 6 times 4 is 24.

128 4 1 6
4

4|0)51|3 4 6 0

12 33

Anſwer 12 tuns, 33 feet, 4 inches, 6 parts.

Note, That 40 feet of timber is a tun, and 50 feet a load.

Note alſo, That 4 feet broad 4 feet deep, and 8 feet long, is a cord of firewood, that is, 128 cubical feet.

Queſt. 7. There is a cellar to be dug by the floor, whoſe length is 33 feet 7 inches, and the breadth is 18 feet 9 inches, and its depth is to be 5 feet 9 inches; I demand how many floors of earth are in that cellar?

	F.	I.		
	33	7		The length.
	18	9		The breadth.
	264			
	33			
	16	9	6	
	8	4	9	
	9	0	0	
	1	6	0	
	629	8	3	
	5	9		The depth.
	3148	5	3	
	314	10	1	
	157	5	0	
324)	3620	8	4(11	
	380			
	56			

Anſwer. 11 floors, 56 feet.

Note, That 18 feet ſquare and a foot deep is a floor of earth, that is 324 ſolid feet.

Queſt. 8. There is a roof covered with tiles, whoſe depth on both ſides (with the uſual allowance at the eaves) is 35 feet 6 inches, and the length 48 feet 9 inches; how many ſquares of tiling are contained therein?

F. I.
48 9
35 6

240
144
24 4 6
17 6 0
8 9 0

17|30 7 6 *Anſw.* 17 ſquares, 30 feet.

Queſt. 9. There is a cone, whoſe diameter at the baſe is 42 inches, and the perpendicular height 94 inches, and it is required to cut off two ſolid feet from the top end thereof; I demand what length upon the perpendicular muſt be cut off?

42
42

84
168

1764
.7854

7056
8820
14112
12348

1385.4456
94

55417824
124690104

3)130231.8864

.43410.628

1722
2

3456

94
94

376
846

8836 Square.
94

35344
79524

830584 The cube

All the ſolid bodies are in triplicate reaſon of their homologous ſides by *Eucl.* 12, 12; 12, 18; and 11, 33; therefore it will be,

	Sol. of the Cone.		Cube Alt.		Sol of 2 Feet.
As	43410.6288	:	830584	: :	3456:

3456

4983504
4152920
3322336
2491752

43410.6288)2870498304(66124 The cube of the length.
• • • •

26580576
5396803
1055740
187528

• •
66124 (40.43
64

2124000 Reſolvend.

12
48

492 Diviſor.

120
4800

48120 Diviſor.

64
1920
19200

1939264 Subtrahend.

184736000 Reſolvend.

1212
489648

4897692

27
10908
1468944

147003507 Subtrahend.

37732493

Anſwer. The length upon the perpendicular muſt be 40.43 inches. If it had been 3 feet, the length had been 46.29 inches.

If two feet were to be cut off from the bottom, or greateſt end, then from 43410.6288 ſubtract 3456, and the remainder is 39954.6288. Then ſay,

As 43410.6288 : 830584 :: 39954.6288
830584

1598185152
3196370304
1997731440
1198638864
3196370304

13410.6288)33185675407.2192(764459)91.4
..... 729
279823524
19359751 35459
1995500
259075 27
42022 243

2952 2457

2457

271
243

24571

10888000

273
24843

248703

Anſwer. It muſt be cut at 91.4 inches from the top, or 2.6 inches from the bottom.

Queſt. 10. If a ſquare piece of timber be 12 feet long, and if the ſide of the ſquare of the greater baſe be 21 inches, and the ſide of the ſquare of the leſſer baſe be 3 inches; how far muſt I meaſure from the greater end to cut off 5 ſolid feet?

Firſt, find the length of the whole pyramid, thus; the difference between 21 and 3 is 18; then,

Diff. Length great. Length.
As 18 : 12 :: 21 : 14

So I find the whole length of the pyramid 14 feet or 168 inches.

The ſolid content of the whole pyramid is 24696 inches, and the ſolid content of 5 feet is 8640; which ſubtracted from 24696, there remains 16056 inches. Then, the cube of 168 (the length) is 4741632. Then,

As 24696 : 4741632 :: 16056 :

To 3082752, whoſe cube root is 145.54; ſubtract this root from 168 (the length) and there remain 22.46 inches, which is the length of 5 ſolid feet at the greater end.

Queſt. 11. Three men bought a grinding-ſtone of 40 inches diameter, which coſt 20 ſhillings; of which ſum the firſt man paid 9 ſhillings, the ſecond 6 ſhillings, and the third 5 ſhillings; I demand how much of the ſtone each man muſt grind down, proportionable to the money he paid?

All circles are in duplicate reaſon of their diameters by *Eucl.* 12.2.

Square the ſemidiameter, which makes 400. Then.

$$\text{And as } \overset{s.}{20} : 400 :: \overset{s.}{9} : 180.$$

This 180 is the ſquare of the ſemidiameter of the circle belonging to the firſt man.

$$\text{And } \overset{s.}{20} : 400 :: \overset{s.}{6} : 120.$$

This 120 is the ſquare of the ſemidiameter of the circle belonging to the ſecond.

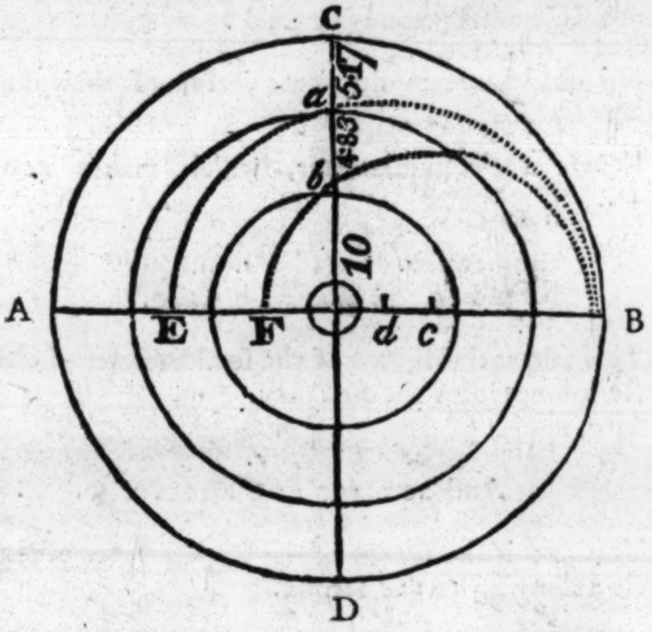
$$\text{And, as } \overset{s.}{20} : 400 :: \overset{s.}{5} : 100.$$

This 100 is the ſquare of the ſemidiameter of the circle belonging to the third.

Then, from 400 (the ſquare of the ſemidiameter of the ſtone) ſubtract 180, and there remain 220, whoſe ſquare root is 14.83 inches; which ſubtracted from 20 inches (the ſemidiameter) there remain 5.17 inches, which is the breadth of the ring, or part of the ſtone which muſt be ground down by the firſt.

Then, from 220 ſubtract 120, and there remain 100, whoſe ſquare root is 10: ſubtract that from 14.83, and there remain 4.83 inches, the breadth of the ring, or part to be ground down by the ſecond man. The third muſt grind down the remainder, which is 10 inches, the ſquare root of 100.

This queſtion may very eaſily and ſpeedily be performed geometrically, as in the annexed ſcheme,



Firſt, upon the center ⊙ ſtrike the circle ACBD and croſs it at right angles with the two diameters AB and CD: then divide the ſemi-diameter A ⊙ (which ſuppoſe 20) in proportion, to 9*s*. 6*s*. and 5*s*. (the ſeveral ſums paid by the three men) by the points E and F; ſo ſhall AE be 9, EF 6 and F⊙5: then divide EB into two equal parts in *d*, and upon *d*, as a center, ſtrike the ſemi-circle E*a*B, and divide FB into two equal parts in *c*, and upon *c*, as a center with the radius *c*F, ſtrike the ſemi-circle F*b*B: ſo have you the ſemi-diameter ⊙ C divided into three ſuch parts as the ſtone ought to be divided; and circles ſtruck through theſe points will ſhew how much each man muſt grind for his ſhare.

Queſt. 12. A gard'ner he had an upright cone,
Out of which ſhould be cut him a rolling ſtone,
The biggeſt that e'er it could make:
The maſon he ſaid, that there was a rule
For ſuch ſort of work, but he had a thick ſkull
Now help him for pity's ſake.

Anſwer. It muſt be cut at one third part of the altitude.

Queſt. 13. There is a ciſtern whoſe depth is ſeven tenths of the wedth, and the length is ſix times the depth, and the ſolid capacity is 367.5 feet; I demand the depth, width and length, and how many buſhels of corn it will hold?

Firſt, you muſt find three numbers in proportion to the depth, width and length, thus; ſuppoſe the depth 7, then the width will be 10, and the length 42; which multiplied together, the product is 2940, which are the ſolid inches in a ciſtern, whoſe depth is 7, width 10, and length 42. But the ſolid inches in the queſtion are 635040(=367.5 × 1728) then the cube of the ſuppoſed width is 1000. So it will be,

As 2940 : 1000 : : 635040 : 216000, whoſe cube root is 60, which is the true width; 7 tenths thereof is 42, the depth; and 6 times 42 is 252 inches the length; which three numbers being multiplied together the product will be 635040. If theſe ſolid inches be divided by 2150.42, the quotient is 295 $\frac{66610}{215042}$ buſhels, or 36 quarters, 7 buſhels, 1 peck, 4 pints. And ſo much will the ciſtern hold.

Queſt. 14. Suppoſe, Sir, a buſhel be exactly round.
Whoſe depth, being meaſur'd, 8 inches is found,
If the breadth 18 inches and half you diſcover,
This buſhel is legal all *England* over.
But a workman would make one of another frame,
Seven inch and a half muſt be the depth of the ſame:
Now, Sir, of what length muſt the diameter be,
That it may with the former in meaſure agree?

```
           18.5
           18.5
          -----
            925
          1480
          185
          ------
         342.25  the ſquare.
          .7854
       ---------
         136900
        171125
       273800
      239575
      ----------
      268.803150
               8
      -----------
     2150.425200 the ſolid inches in a buſhel.
              7.5) 2150.4252 (286.72336
                    .   .   .
                   ----------
                   650
                     504
                      542
                        175
                         252
                          270
                           450
                          ----
                             0
```

.7854)286.72336 . . .

51103
39763
52360

5236

. . . .
)365.0666(19.107
1

29)265
261

381)406
381

38207)256.600

Anſwer. The diameter muſt be 19.107 inches, if the depth be 7.5 inches.

Queſt. 15. *In the midſt of a meadow well ſtored with graſs,*
I took juſt an acre to tether my aſs:
How long muſt the cord be that feeding all round,
He mayn't graze leſs nor more than his acre of ground?

By *Problem* 10. Section 9. chap. 1. find the diameter of a circle containing an acre; half that will be the length of the cord.

The Work.

660 feet, the length of an acre.
60 feet, the breadth of an acre.

3960
3960

43560 the ſquare feet in an acre.

As 1 : 1.2732 :: 43560
43560

763920
6,660
38196
50928

. . .
55460.5920(235.5 Diamet.
4 117.75 Half.
43)154
129
465)2560
2325
4705)23559
23525
34

Anſwer. The cord muſt be 117 feet and 9 inches.

But in an *Iriſh* acre is 70560 feet, *i. e.* 21×21×160=70560; then ſay as above,

As 1 : 1.2732 : : 70560
70560
763920
63660
891240

. . . . .
89836.9920(299.72 Diamet.
4 149.86 Half.
49) 498
441
589) 5736
5301
5987) 43599
41909
59942) 169020

Queſtion 16. A malſter has a kiln that is 16 feet 6 inches ſquare; but he is minded to pull it down and build a new one, that may be big enough to dry three times as much at a time as the old one will do; I demand how much ſquare the new one muſt be?

```
        16.5
        16.5
      ------
         825
        990
       165
      ------
      272.25 the area of the old one.
           3
      ------
      .  .  .
      816.75(28.57
      4
      ------
   48)416
      384
      ------
  565)3275
      2825
      ------
 5707)45000
      39949
      ------
       5051
      ------
```

Anſwer. The ſide of the new one muſt be 28 feet and near 7 inches.

Queſtion 17. If a round ciſtern be 26.3 inches diameter, and 52.5 inches deep, how many inches diameter muſt a ciſtern be, to hold twice the quantity, the depth being the ſame? and how many ale gallons will each ciſtern hold?

26.3
26.3
789
1578
526
691.69 the ſquare.
2
. . .
1383.38(37.19
9
67)483
469
741)1438
741
7429)69700
66861
2839

The diameter of the greater is 37.19 inches.

```
      691.69 the ſquare of the leſſer ciſtern's
       .7854                            [diam.
    --------
      276676
     345845
    553352
   484183
    --------
   543.253326 Area of the baſe.
         52.5
   ----------
   2716266630
  1086506652
 2716266630
 -------------
 28520.7996150 Solid content in inches.
 -------------
 282)28520.799(101 137 Gallons.
     ---------
      320
       387
       1059
        2139
      ------
        165
```

NOTE, That 282 ſolid inches is an ale or beer gallon, and 231, a wine gallon.

And 359.05 is the ſquare of the diameter of a circle that will hold a gallon of ale at an inch deep, and 294.12 for wine.

And 217.6 inches is an *Iriſh* gallon, of either ale or wine. Alſo 277.05 is the ſquare of the diameter of a circle, that will hold an *Iriſh* gallon at an inch deep.

You may find the content in gallons, thus, divide the ſquare of the diameter by 359 05, and multiply the quotient by the depth.

359.05)1383.38 . . .	(3.853
	52.5
306230	
18990	19265
1037	7706
	19265

The content of the greater 202.2825 Gallons.

Queſtion 18. If the diameter of a caſk at the bung be 32 inches, and at the head 25 inches, and the length 40 inches, how many ale gallons are contained therein?

25	32	
25	32	359
		3
125	64	
50	96	1077
625	1024 Square of the bung diameter.	

1024
625 Square of the head diameter.

1077)2673	(2.48	But for *Iriſh* gallons,	
	40	Divide by	831.15
5190		The quotient is	3.21
8820	99.20	Which multiply by	40
204		Give *Iriſh* gallons	128.40

Anſw. 99.2 gallons.

Otherwiſe, you may find a mean diameter, and work by ſcale and compaſſes, thus: ſubtract 25 from 32, and there remain 7, which multiplied by .7, the product is 4.9, which added to 25, the ſum is 29.9. Then extend the compaſſes from 18.95, the gage point *Engliſh*, to 29.9, that extent turned twice from 40 (the length) will fall upon 99.6 gallons ſomething more than before.

Queſtion 19. There is a ſtone 20 inches long, 15 inches broad, and 8 inches thick, which weighs 217 pounds; I demand the length, breadth, and thickneſs of another of the ſame kind and ſhape, which weighs 1000 pounds?

The Cube of 20 (the length) is 8000. Then, (by *Euclid*, 11. 33.)

As 217 : 8000 : : 1000 : 36870.645, whoſe cube root is 33.28 inches, the length of the ſtone weighing 1000 pounds. Then ſay,

As 20 : 33.28 : : 15 : 24.96
As 20 : 33.28 : : 8 : 13.312

Anſwer { The length 33.28
The breadth 24.96
The thickneſs 13.312 } Inches.

Queſtion 20. If an iron bullet, whoſe diameter is 4 inches, weighs 9 pounds, what will be the weight of another bullet (of the ſame metal) whoſe diameter is 9 inches?

The cube of 4 is 64, and the cube of 9 is 729, then, (by *Eucl.* 12. 18.)

℔ ℔
As 64 : 9 : : 729 : 102.515
℔ *oz.* *dr.*
Anſwer. It weighs 102 8 4 *fere.*

Queſtion 21. There is a ſquare pyramid of marble, each ſide of its baſe is 5 inches, and the height thereof 15 inches, and its weight is 12 pounds and a quarter; I demand the weight of another like ſquare pyramid, each ſide of whoſe baſe is 30 inches?

The cube of 5 is 125, and the cube of 30 is 27000. Then, (by *Eucl.* 12. 12.)

℔ ℔
As 125 : 12.25 : : 27000 : 2646
Anſwer. The weight is 2646 pounds.

Queſtion 22. There is a ball or globe of marble, whoſe diameter is 6 inches, and its weight 11 pounds; what will be the diameter of another globe of the ſame marble, that weighs 500 pounds?

The cube of 6 is 216. Then,

```
        ℔               ℔
   As 11 : 216 : : 500 : 9818.1818.
```

whoſe cube root is 21.4 inches, the diameter ſought.

Queſtion 23. There is a fruſtum of a pyramid, whoſe baſes are regular octagons; each ſide of the greater baſe is 21 inches, and each ſide of the leſſer baſe is 9 inches, and its length is 15 feet; I demand how many ſolid feet are contained therein?

```
     4.8284 the tabular number, page 95.
        237 the ſquare of a mean ſide.
     ------
     337988
    144853           21           12
    96568             9           12
  ---------        ----         -----
  1144.3308         189        3) 144
         15          48         -----
  ---------        ----            48
   57216540         237
  11443308
  ---------
144)17164.9620(119.2
    ----------
  276
  1324
   289
     1
```

Anſwer. 119.2 ſolid feet.

Queſtion 24. There is a fruſtum of a cone, the diameter of the greater baſe is 36 inches, and the diameter of the leſſer baſe is 20 inches, and the length or height is 215 inches; I demand the length and ſolid content of the whole cone, and alſo the ſolid content of the given fruſtum?

Firſt find the length of the whole cone, thus:

From 36
Subtr. 20

As 16 : 215 : : 36 : 483.75

ſo the length of the whole cone is 483¾ inches.

Then find the content of the whole cone.

36
36

216
108

1296
.7854

5184
6480
10368
9072

Area baſe 1017.87 84

1017.8784
52.161

10178784
6107270
101788
20357
5089
Feet.
1728)164132.88(94.98

8612
17008
14568

744

Thus I find the ſolidity of the whole cone 94.98 feet.

Then find the ſolid content of the top part that is wanting.

.7854

.7854 the area of unity.
400 the ſquare of 20.

3)314.1600 area of the leſſer baſe.

104.72 a third part.
268.75 altitude of the top part.

52360
73304
83776
62832
20944

1728)28143.500(16.28 feet.

10863
4955
14990

1166

	Feet.
Content of the whole	94.98
Content of the top piece	16.28
Content of the fruſtum	78.7

Queſtion 25. If the top part of a cone contains 26171 ſolid inches, and 200 inches its length, and the lower fruſtum thereof contains 159610 ſolid inches; I demand the length of the whole cone, and the diameter of each baſe?

200	159610 } add
200	26171 }
40000	185781 the ſum.
200	
8000000	

As 26171 : 8000000 :: 185781 : to 56789881, whoſe cube root is 384.3 inches, the length of the whole cone.

Then find the diameter of the leſſer baſe, thus;
200)26171
130.855
3
392.565 area of the leſſer baſe.

Then, by Prob. 10 Sect. IX. Chap. I.
As 1 : 1.2732 :: 392.565
1.2732
785130
1177605
2747955
785130
392565
499.8137580(22.35
4
42)99
84
443)1581
1329
4465)25237
22325
2912

	Leſſer leng.	leſſ. diam.	great. leng.	gr. diam.
Again, As	200 :	22.35 ::	384.3 :	4294.

		Inches.
Anſw.	The length of the whole cone	384.3
	The diameter of the greater baſe	42.94
	The diameter of the leſſer baſe	22.35

Queſtion 26. There is a fruſtum of a cone whoſe ſolid content is 20 feet, and its length 12 feet; and the greater diameter bears ſuch proportion to the leſſer as 5 to 2; I demand the diameters.

$5\times5=25$
$2\times2=\ \ 4$
$5\times2=10$
The ſum 39

$3)12$
$4)20(5$ Feet.
Theſe 5 feet are the triple of a mean area.

Then, as 1 : 1.27324 : : 5 : 6.3662.

So the triple ſquare of a mean diameter is 6.3662.

Then, as 39 : 6.3662 : : 25 : 4.080897.

This 4.080897 is the ſquare of the greater diameter, whoſe ſquare root is 2.020123 feet which is 24.24147 inches.

Then, as 5 : 24.24147 : : 2 : 9.69659.

So the greater diameter is 24.24147. the leſſer diameter is 9.69659 inches.

Queſt. 27. There is a room of wainſcot 129 feet 6 inches in circumference, and 16 feet 9 inches high, (being girt over the mouldings) there are two windows each 7 feet 3 inches high, and the breadth of each from cheek to cheek, 5 feet 6 inches; the breadth of the ſhutters of each is 4 feet 6 inches: the cheek-boards and top and bottom boards of each window taken together, is 24 feet 6 inches, and their breadth 1 foot 9 inches; the door caſe 7 feet high, and 3 feet 6 inches wide; the door 3 feet 3 inches wide; I demand how many yards of wainſcot are contained in that room?

F.	I.		
129	6		
16	9		
782	0		
129			
64	9		
32	4	6	
2196	1	6	

F.	I.		
3	3		
7	0		
22	9		
11	4	6	Half.
34	1	6	

F.	I.		
7	3		
5	6		
36	3		
3	7	6	
39	10	6	
		2	
79	9	0	

F.	I.		
7	3		
4	6		
29	0		
3	7	6	
32	7	6	
16	3	9	Half.
48	11	3	
		2	
97	10	6	

F.	I.		
24	6		
1	9		
24	6		
18	4	6	
42	10	6	
		2	
85	9	0	

F.	I.	
3	6	
7	0	
24	6	add.
79	9	
104	3	

The content of the room	2169	1	6
The ſhutters, at work and half	97	10	6
The door, at work and half	34	1	6
The check-boards, *&c.*	85	9	0
The ſum	2386	10	6
The window-lights and door-caſe deduct	104	3	0
	9) 2282	7	6
Anſw. 253 yards, 5 feet.	253	5	

Queſt. 28. There is a wall which contains 18225 cube feet, and the height is 5 times the breadth, and the length 8 times the height; what is the length, breadth and height?

Suppoſe the breadth 2, then the height muſt be 10, and the length 80; which three nnmbers multiplied together, the product will be 1600, and the cube of 2 is 8: then ſay,

As 1600 : 8 :: 18225 : 91.125.

Then the cube root of 91.125 is 4.5, which is the breadth; then 5 times 4.5 is 22.5, the height; and 8 times 22.5 is 180, the length.

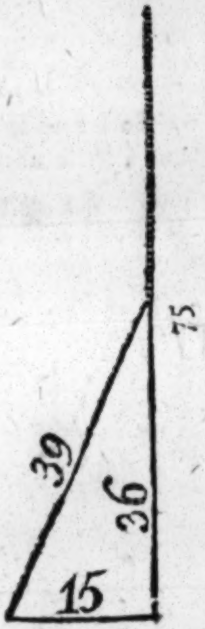
Queſt. 29. There is a may-pole, whoſe top-end was broken off by a blaſt of wind, and the top-end in falling ſtruck the ground at 15 feet diſtance from the foot of the may-pole; the broken piece was 39 feet; now I demand the length of the may-pole?

By *Eucl.* 1. 47, the ſquare of the hypothenuſe of a right-angled triangle, is equal to the ſum of the ſquares of the baſe and perpendicular.

Therefore, from the ſquare of 39 ſubtract the ſquare of 15, the ſquare root of the remainder is the piece ſtanding, to which add the piece broke off, and you have the whole length.

39	15
39	15
351	75
117	15
1521	225
225	

. .
1296(36
9
66)396
396
...



The piece ſtanding is 36 feet.
The piece broken off is 39 feet.
The whole length 75

Queſt. 30.

A may-pole there was, whoſe length I would know;
The ſun ſhining clear, ſtraight to work I did go:
The length of the ſhadow, upon level ground,
Juſt ſixty-five feet, when meaſured I found:
A ſtaff I had there, juſt five feet in length;
The length of its ſhadow was four feet one tenth:
How high was the may-pole, I gladly would know,
And it is the thing you're deſir'd to ſhew.

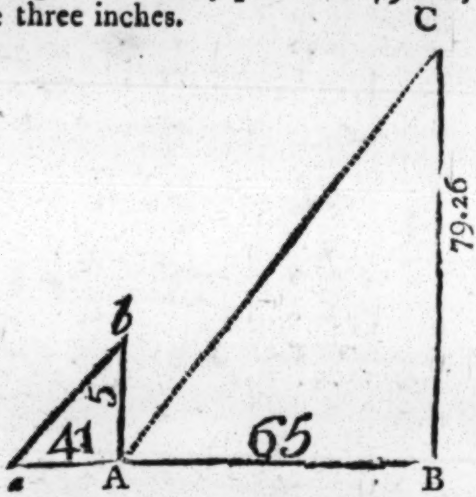
By *Eucl.* 6. 4.

As aA : aAb : : AB : BC.

That is,

As 4.1 : 5 : : 65 : 79.26.

So I find the height of the may-pole to be 79 feet, and a little above three inches.



Here AB repreſents the length of the ſhadow of the may-pole, and BC the may-pole; *a*A the ſhadow of the ſtaff, and A*b* the ſtaff.

Queſt. 31. What will be the diameter of a globe, when the ſolidity and ſuperficial content thereof are equal?

If the diameter be 1, the ſolidity will be .5236, and the ſuperficies will be 3.1416; that is, as 1 to 6. And to find the ſuperficial content, we muſt multiply 3.1416 by the ſquare of the axis or diameter, and the product is the ſuperficial content. And for the ſolidity, multiply .5236 by the cube of the axis, the product is the ſolid content; therefore, becauſe .5236 is a ſixth part of 3.1416, we muſt take ſix for the diameter ſought. For if 3.1416 be multiplied by the ſquare of 6, *viz.* by 36, the product will be 113.0976; and if .5236 be multiplied by the cube of 6, *viz.* by 216, the product is likewiſe 113.0976, the ſolidity equal to the ſuperficies.

Therefore, 6 is the true anſwer.

Queſt. 32. What will the axis of a globe be, when the ſolidity is in proportion to the ſuperficies, as 18 to 8?

Becauſe the ſolidity and ſuperficies is as 1 to 6; when the axis of the globe is 1, it will be

As 8 : 18 : : 6 : 13.5.

So the diameter ſought is $13\frac{1}{2}$.

If the proportion of the ſolidity to the ſuperficies had been as 8 to 18, then it will be

As 18 : 8 : : 6 : $2\frac{2}{3}$.

So then the diameter will be $\frac{2}{3}$.

The reaſon of theſe operations, both in this and the laſt Queſtion, is from Algebra.

Queſt. 33. There are three Grenade ſhells of ſuch capacity, that the ſecond ſhell will juſt lie in the concavity of the firſt, and the third in the concavity of the ſecond. The ſolidity of the metal of the firſt ſhell is equal to its concavity; and the ſolidity of the metal of the ſecond, to the concavity, is as 7 to 5; and the ſolidity of the third or leaſt ſhell's metal, to its concavity, is as 9 to 4. Now ſuppoſing the diameter of the firſt, or greateſt ſhell, to be 16 inches, and allowing every ſolid inch of iron to weigh four ounces; I demand the diameter of the two leſſer ſhells, and the thickneſs and ſolidity of metal of every ſhell, and alſo the weight of every ſhell.

The cube of 16 is 4096: then,

As 1 : .5236 : : 4096 : 2144.6656.

The half thereof is 1072.3328, which is the ſolidity of the metal of the greater ſhell, as alſo of the concavity.

As .5236 : 1 : : 1072.3328 : 2048.

The cube root of 2048 is 12.699, which is the diameter of the ſecond ſhell.

The ſum of 7 and 5 is 12; then,

As 12 : 5 : : 1072.3328 : 446.805.

This 446.805 is the ſolid content of the concavity of the ſecond.

As .5246 : 1 : : 446.805 : 853.333.

The cube root of 853.333 is 9.485, the diameter of the leaſt ſhell.

The ſum of 9 and 4 is 13; then,

As 13 : 4 : : 446.805 : 137.47846.

This 137.47846 is the ſolid content of the concavity of the third.

As .5236 : 1 : : 137.47846 : 262.5639.

The cube root of 262.5639 is 6.4034, the diameter of the leaſt ſhell's concavity.

From 16 the diameter of the greateſt.
Subtr. 12.699 the diameter of the ſecond.

Rem. 3.301

Half is = 1.65 the thickneſs of metal of the great.

From 12.699 the diameter of the ſecond.
Subtr. 9.485 the diameter of the leaſt.

Rem. 3.214

Half is = 1.607 the thickneſs of metal of the ſecond.

From 9.485 the diameter of the leaſt.
Subtr. 6.403 the diameter of the concavity.

Rem. 3.082

Half is = 1.541 the thickneſs of metal of the leaſt.

The metal of the greateſt is 1072.33 ſolid inches; which divide by 4 (becauſe every ſolid inch is a quarter of a pound) the quotient is 268.08 pounds.

The metal of the ſecond is 625.52 ſolid inches; which divided by 4, the quotient is 156.38 pounds, the weight of the ſecond.

The metal of the leaſt ſhell is 309.32 ſolid inches; which divided by 4, the quotient is 77.33 pounds, the weight of the leaſt.

The diameter of the	ſecond ſhell	12.699	inches.
	leaſt ſhell —	9.485	

The thickneſs of metal of the	greateſt —	1.65	inches.
	ſecond —	1.607	
	leaſt —	1.541	

The weight of the	greateſt —	268.08	pounds.
	ſecond —	156.38	
	leaſt —	77.33	

Additional Queſtions in SUPERFICIAL MEASURING.

Queſtion 1. WHAT difference is there between a floor, 28 feet long, by 20 broad, and two others, that meaſure 14 feet a piece by 10; and what do all three come to, at 45*s.* *per* ſquare, *viz.* 10 feet by 10?

Anſwer, 280 ſq. feet. Diff. amount 18*l.* 18*s.*

Queſt. 2. If my Court-yard be 47 feet 7 inches ſquare, and I have laid a foot way of *Purbec* ſtone, 4 feet wide, along one ſide of it: what will paving the reſt, with flints, come to, at 6*d.* *per* yard ſquare?

Anſw. 5*l.* 15*s.* 2$\frac{1}{2}$*d.*

Queſt. 3. An elm plank is 14 feet 3 inches long, and I would juſt have a yard ſquare ſlit off: at what diſtance from the edge, muſt the line be ſtruck?

Anſw. 7 inches $\frac{90}{171}$.

Queſt. 4. I have a wooden trough, that at 6*d.* per yard, coſt me 3*s.* 2*d.* painting within; the length of it was 102 inches, the depth 21 inches: what was its breadth?

Anſw. 2 feet, 3$\frac{1}{4}$ inches.

Queſt. 5. Having a fiſh pond of a triangular form; whoſe three ſides meaſure 400 yards, 348, and 312: what quantity of ground does it cover?

Anſw. 52284 $\frac{1}{2}$ ſquare yards.

Queſt. 6. The height of an elm, growing in the middle of a circular iſland, 30 feet in diameter, plumbs 53 feet, and a line ſtretched from the top of the tree, ſtraight to the hither edge of the water, 112 feet: what then is the breadth of the moat, ſuppoſing the land on either ſide the water to be level?

Anſw. $83\frac{2}{3}$ feet.

Queſt. 7. Suppoſe a light-houſe built on the top of a rock; the diſtance between the place of obſervation, and that part of the rock level with the eye, and directly under the building, is given 310 fathoms; the diſtance from the top of the rock, to the place of obſervation, is 423 fathoms; and from the top of the building 425: the height of the edifice is required?

Anſw. 17 feet, 7 inches, nearly.

Queſt. 8. A ladder 40 feet long, may be ſo planted, that it ſhall reach a window 33 feet from the ground, on one ſide the ſtreet; and without moving it at the foot, will do the ſame by a window 21 feet high, on the other ſide: the breadth of the ſtreet is required?

Anſw. $56\frac{6}{10}$ feet.

Queſt. 9. The paving of a triangular court, at 18d. per foot, came to 100l. the longeſt of the three ſides was 88 feet: what then was the ſum of the other two equal ſides?

Anſw. 106.85 feet.

Queſt. 10. There are two columns in the ruins of *Perſepolis*, left ſtanding upright; one is 64 feet above the plane, the other 50: between theſe, in a right line, ſtands an antient ſtatue, the head whereof is 97 feet from the ſummit of the higher, and 86 feet, from the top of the lower column; the baſe whereof meaſures juſt 76 feet to the center of the figure's baſe: by theſe notices, the diſtance of the top of the columns may be, by numbers, eaſily found?

Anſwer, 157 feet, nearly.

Queſt. 11. The ſurveying wheel is ſo contrived, as to turn juſt twice in the length of a pole, or $16\frac{1}{2}$ feet: what then is its diameter?

Anſwer, 2.626 feet.

Queſt. 12. I would turf a round plot, meaſuring 130 feet about, and would know the charge at 4d. per yard ſquare?

Anſwer, 2l. 9s. 10d. nearly.

Queſt. 13. It is obſerved that the extreme end of the minute hand of a public dial, moves juſt 5 inches in the ſpace of $3\frac{1}{4}$ minutes: The queſtion is, what is the length of that index?

Anſwer, 14.69 inches.

Queſt. 14. A pipe of 6 inches bore will be 3 hours in running off a certain quantity of water: what time will 4 pipes each 3 inches bore, be in diſcharging double the quantity?

Anſwer, 6 hours.

Queſt. 15. A yard of rope 9 inches round weighs, ſuppoſe, 22lb: what will a fathom of that weigh, which meaſures a foot about?

Anſwer, $78\frac{2}{9}$ lb.

Queſt. 16. If 20 feet of iron railing ſhall weigh half a ton, when the bars are an inch and quarter ſquare, what will 50 feet of ditto come to at $3\frac{1}{2}$d. per pound, the bars being but $\frac{7}{8}$ of an inch ſquare?

Anſwer, 20*l.*

Queſt. 17. A looking-glaſs is 16 inches by 9, and contains a foot of glaſs: What will the content of the plate be that has twice the length, and three times the breadth?

Anſwer, 6 ſquare feet.

Queſtions in SOLID MEASURING.

Queſt. 1. A Priſm of two equal baſes, and ſix equal ſides, that meaſures 28 inches croſs the center from corner to corner: the ſuperficial and the ſolid content is required, taking the length at 134 inches?

Anſwer, Superf. 9 yards, 4 feet, 3 inches.
Solid 39 feet, 843 inches

Queſtion 2. I have a rolling ſtone 44 inches in circumference, and am to cut off three cubic feet from one end: Whereabouts muſt the ſection be made?

Anſwer, At 33.66 inches.

Queſt. 3. Suppoſing the atmoſphere, or body of the air and vapours ſurrounds the the globe of the earth and ſea, to 60 miles above the ſurface; the earth is 7970 miles in diameter; how many cubic yards of air then hang about, and revolve along with this planet: and what is the weight of the whole maſs of fluid matter in the atmoſphere contained if at a medium 12 ℔ *Avoirdupois*, be found experimentally, as in fact it is, to preſs upon every circular inch, on the ſurface of the earth?

Anſwer, Meaſure 6626425427414876l600 ſolid yards, weight 12240191636766720000 pounds *Avoirdupois*.

Queſt. 4. A cork may be cut into ſuch a form, that it may, without alteration, ſeverally fill the cavity of a circle an inch in diameter, of an equilateral triangle, whoſe ſides are each an inch, and a geometrical ſquare, alſo an inch, on a ſide. What muſt be the ſhape?

Anſwer. It will be a wedge, whoſe baſe, being circular, will fill the round: the ridge-front will repleniſh the ſquare, and the end-ſection make out the triangle.

Queſt. 5. Let it be demanded to find the ſide and ſolid content of the greateſt cube that can be contained in a globe of 27 inches diameter.

Queſt. 6. If a piece of timber 48 feet long, 6 inches broad, and 14 inches thick, coſt 10*l.* what is the value of 5 ſuch pieces which are 8 feet long, 4 inches broad, and 6 inches thick?

Queſt. 7. If 300 men in 15 days dig a trench 5600 feet long, 6 feet deep, and 12 wide; how long muſt that trench be whoſe depth is 8 feet, and width 14 feet, dug by 2700 men in 25 days?

Queſt. 8. If 24 men working 189 days 12 hours each day, dig a trench $33\frac{1}{2}$ yards long, $3\frac{1}{2}$ deep, and $5\frac{2}{3}$ wide; how many hours per day muſt 217 men work to dig a trench 23 yards long, $2\frac{1}{2}$ deep, and $3\frac{1}{3}$ wide in 6 days?

Queſt. 9. A Bath-ſtone, 20 inches long, 15 over, and 8 deep, weighs 220lb. How many cubit feet thereof will freight a ſhip of 290 ton?

A SHORT

A SHORT

APPENDIX.

§ I. *Of* GAGING, *according to both the* Iriſh *and* Engliſh *Gallon.*

I SHALL not here give the whole art of gaging, (there being ſeveral books of that art already in print, wrote by better hands) but ſhall only lay down ſome ſhort practical rules whereby any artificer or others, may find the quantity of liquor in any veſſel, upon occaſion.

PROBLEM I.

To find the ſeveral Multipliers, Diviſors and Gage-points, belonging to the ſeveral Meaſures now uſed in England *and* Ireland.

282)1.0000(.003546 Multiplier for ale gallons.
231)1.0000(.004329 Multiplier for wine gallons.
268.8)1.000(.0037202 Multiplier for corn gallons
2150.42)1.000(.00046502 Multip. for corn buſhels
217.6)1.0000(.0045955 Multip. for *Iriſh* gallons

So, if the ſolid inches in any veſſel be multiplied by the ſaid multipliers the product will be gallons in the reſpective meaſures: or, dividing by the diviſors 282.231, 268.8, or, 217.6, the quotients will likewiſe be gallons.

NOTE, That 282 ſolid inches is a gallon of ale or beer meaſure; 231 ſolid inches is a gallon of wine meaſure; 368.8 ſolid inches is a gallon, and 2150.42 ſolid inches is a buſhel of corn-meaſure. Alſo 217.6 ſolid inches is a gallon *Iriſh* both of ale, wine, or oil.

For circular areas, the following multipliers and diviſors are to be uſed.

282).785398(002785 Multiplier for ale gallons.
231).785398(00339 Multiplier for wine gallons.
2176).785398(0036093 Multiplier for *Iriſh* gallons
.785398)282(359.05 Diviſor for ale gallons.
.785398)231(294.12 Diviſor for wine gallons.
.785398)2150.42(2738 Diviſor for corn buſhels.
.785398)217.6(277.05 Diviſor for *Iriſh* gallons.
The ſquare root of the diviſor is the gage-point.

The gage-point for ſquares in	Ale meaſure,	is	16.79
	Wine meaſure,		15.19
	Malt buſhel,		46.36
	Iriſh gallon,		14.75

The gage-point for circular figures in	Ale meaſure,	is	18.95
	Wine meaſure,		17.15
	Malt buſhel,		52.32
	Iriſh gallon,		16.64

PROBLEM II.

To find the Area in Ale, Wine, or Iriſh *Gallons, of any rectilineal plane Figure, whether Triangular, Quadrangular, or Multangular.*

TO reſolve this *problem*, you muſt, by Chap. I. Part II. find the area in inches and then bring it into gallons, by dividing that area in inches by the proper diviſor, *viz.* by 282 for ale, or by 231 for wine, or 217.6 for *Iriſh*; or elſe by multiplication, by .003546 for ale, by .004329 for wine, by .0045955 for *Iriſh*, and the quotient or product will be the area.

Example. Suppoſe a back or cooler in the form of a parallelogram, or long ſquare, 250 inches in length, and 84.5 inches in breadth: what is the area in ale, wine, or *Iriſh* gallons?

Multiply 250 by 84.5, and the product is 21125 the area in inches; which divided by 282, and the quotient is 74.9 gallons of ale; or multiplied by .003546, the product is 74.90925 gallons, nearly the ſame; and if 21125 be divided by 231, or multiplied by .004329, it will give 91.45 gallons of wine; and if 21125 be divided by 217.6, or multiplied by .004595, it will give 97.06 *Iriſh* gallons.

By Scale *and* Compaſſes.

Extend the compaſſes from 282 to 250, that extent will reach from 84.5 to 74.9: And

Extend from 231 to 250, that extent will reach from 84.5 to 91.45.

Extend the compaſſes from 217.6 to 250, that extent will reach from 84.5 to 97.06.

NOTE, The areas of all ſuperficies are always to be underſtood to be one inch deep, otherwiſe it could not be ſaid that the area of ſuch a parallelogram, circle, *&c.* is ſo many gallons.

Having found the area of a back or cooler, the next thing will be to find out the true dipping or gaging-place in that back, that ſo the true quantity of worts may be computed at any depth; which may be thus done.

1. When the bottom of the back is covered all over (of any depth) with worts or other liquor, then dip it in eight or ten ſeveral pieces (more or leſs, according to the largeneſs of the back) as remote and equally diſtant from each other as you can well do, noting down the wet inches and decimal parts of every dip.

2. Divide the ſum of all thoſe dips by the number of places you dipped in, and the quotient will be the mean wet of all thoſe dips.

3. *Laſtly*, find out ſuch a place by the ſide of the back (if you can) that juſt wets the ſame with that mean dip, and make a notch or mark there for the true and conſtant dipping place of that back.

Then, if any quantity of worts (which covers the whole back) be dipped or gaged at that place, and the wet inches ſo taken be multiplied into the area of the back in gallons, the product will ſhew how many gallons of wort are in that back at that time, provided the ſides of the back do ſtand at right-angles with the bottom.

PROBLEM III.

The Diameter of a Circle being given in Inches, to find the Area thereof in Ale, Wine, or Iriſh *Gallons.*

IF the ſquare of the diameter be multiplied by .002785 for ale, or by .003399 for wine, or by .0036093 for Iriſh gallons; or if it be divided by 359.05 for ale, or by 294.12 for wine, or by 277.05 for Iriſh gallons, the products or quotients will be the reſpective ale, wine, or Iriſh gallons.

Example. Suppoſe the diameter of a circle be 32.6 inches, what will be the area in ale, wine, or Iriſh gallons?

The ſquare of 32.6 is 1062.76.

Then 359.05) 1062.76(2.9599 Area in ale gal.
And 294.12) 1062.76(3.6133 Area in wine gal.
And 277.05) 1062.76(3.8565 Area in Iriſh gal.
Or 1062.76×.002785=2.9599 Ale gal.
And 1062.76×.003399=3.6133 Wine gal.
And 1062.76×.003609=3.8565 Iriſh gal.

By Scale *and* Compaſſes.

Extend the compaſſes from 18.95 (the gage-point for ale) to 32.6 (the diameter) that extent will reach from 1 to a fourth number and from that fourth to 2.9599 gallons. Or extend the compaſſes from 1 to 32.6, that extent, turned twice over from .002785 will at laſt fall upon 2.9599.

For wine: extend from 17.15 (the gage-point for wine) to 32.6 (the diameter) that extent turned twice over from 1, will at laſt fall upon 3.6133 gallons.

Or thus: extend from 1 to 32.6, that extent will reach from .003399 (being twice turned over) to 3.6133 wine gallons.

For Iriſh gallons: extend from 16.64 (the Iriſh gage-point) to 32.6 (the diameter) that extent, turned twice over from 1, will at laſt fall upon 3.8565, the Iriſh gallons ſought.

PROBLEM IV.

The tranſverſe (or longeſt Diameter) and the Conjugate or ſhorteſt Diameter, of an Ellipſis (or Oval) being given, to find its Area in Ale, Wine, or Iriſh Gallons.

IF the rectangle, or product of the two diameters, that is, of the length and breadth of the oval, be divided by 359.05, or multiplied by .002785 for ale, or divided by 294.12, or multiplied by .003399 for wine, or divided by 277.05, or multiplied by .0036093 the quotients or products will be the ale, wine, or Iriſh gallons required.

Example. Suppoſe the longeſt diameter be 81.4 inches, and the ſhorteſt diameter be 54.6 inches, what will be the area of that oval?

Multiply 81.4 by 54.6, and the product is 4444.44; then

359.05)4444.44(12.38 Area in ale gal.
294.12)4444.44(15.11 Area in wine gal.
277.05)4444.44(16.71 Area in *Iriſh* gal.
Or 4444.44×.002785=12.38 Ale gal.
And 4444.44×.003399=15.11 Wine gal.
And 4444.44×.003609=16.1 *Iriſh* gal.

By Scale *and* Compaſſes.

Firſt find a mean proportional between 81.4 and 54.6, by dividing the diſtance between them into two equal parts, and the middle point will be at 66.6, which is the mean proportional, that is, the diameter of a circle equal to the oval. Then extend the compaſſes from 18.95 (the gage-point for ale) to 66.6, that extent turned twice over from 1, will at laſt fall upon 12 38 ale gallons: and extended from 17.15 (the gage-point for wine) to 66.6, that extent, turned twice over from 1, will reach at laſt to 15.11 wine gallons.

Laſtly, Extend the compaſſes from 16.64 (the *Iriſh* gage-point) to 66.6, that extent, turned twice over, from 1, will at laſt fall upon 16.1 *Iriſh* gallons.

PROBLEM V.

To find the Content in Ale, Wine, or Iriſh *Gallons, of any Priſm, what Form ſoever its Baſe is of.*

FIRST find its ſolid content in inches (by ſect. 1, 2, 3, of Chap. II. Part II.) then divide that content in inches by 282, for ale, or by 231 for wine, or by 217.6 for *Iriſh*; the reſpective quotients will be the content in wine, ale, or *Iriſh* gallons.

Otherwiſe, you may find the content of a Priſm, by finding the area of its baſe in gallons (by *Problem* II. of this Appendix) and multiply that area by the tun's height or depth within, the product will be its content in gallons.

Example. Suppoſe a tun, whoſe baſe is a parallelogram right-angled, its length being 49.3 inches, its breadth 36.5 inches, and the depth of the tun is 42.6 inches; the content in ale, wine, and *Iriſh* gallons is required.

The length, breadth and depth being multiplied continually, the product is 76656.57; which divided by 282, the quotient is 271.83 ale gallons; and divided by 231, the quotient is 331.84 wine gallons; and divided by 217.6, the quotient is 252.23 *Iriſh* gallons; and by dividing by 2150.4, ſuch a ciſtern will be found to hold 35.65 buſhels of corn.

By Scale *and* Compaſſes.

Extend the compaſſes from 282 to 36.5 (the breadth of the baſe) that extent will reach from 49.3 (its length) to 6.38 ale gallons (the area of the baſe) then extend from 1 to 42.6 (the depth) that extent will reach from 6.38 (the area of the baſe) to 271.8 gallons, the content.

PROBLEM VI.

To find the Content of a Tun, whoſe Baſes are alike and parallel, but unequal, being the Fruſtum of a Pyramid.

FIND the area of each baſe, and a mean proportional between them, and multiply the ſum of thoſe three by one third part of the depth or height, and the product is the content.

Example. Suppoſe a tun whoſe baſes are parallelograms, the length of the greater is 100 inches, and its breadth 70 inches; the length of the leſſer baſe 80, and its breadth 56; and the depth of the tun 42 inches; the content in ale and wine gallons is required.

Multiply 100 by 70, the product is 7000, the area of the greater baſe; and 80 multiplied by 56, the product is 4480, the area of the leſſer baſe; then multiply the two areas into each other, and the product is 31360000, whoſe ſquare root is 5600, a geometrical mean proportional.

The greater area	7000	add
The leſſer area	4480	
The mean proportional	5600	
	17080	
A third of the depth	14	
	68320	
	17080	

282)239120(847.94 Ale gal.
231)239120(1035.15 Wine gal.
217.6)239120(1098.9 Iriſh gal.

PROBLEM VII.

To find the Content of a Tun, whoſe Baſes are parallel and circular, being the Fruſtum of a Cone.

YOU may find the content as in the laſt *problem*, by multiplying the ſum of the areas of the two baſes, and a mean proportional, by one third part of the depth.

But it will be a ſhorter way to find the area of a mean circle in gallons, and multiply that by the depth, thus: To the rectangle of the greater and leſſer diameters, add one third part of the ſquare of the difference of the diameters, that ſum is the ſquare of a mean diameter, which divided by 359.05 for ale, or by 294.12 for wine, or by 277.05 for *Iriſh* gallons; gives the area of a mean circle in ale, wine, or *Iriſh* gallons; which multiplied by the depth, gives the content.

Example. Suppoſe the greater diameter 80 inches, and the leſſer diameter 71 inches, and the depth 34 inches; the content in ale, wine, and *Iriſh* gallons is required.

Multiply 80 by 71, and the product is 5680; to which add 27 (a third part of the ſquare of the difference of the diameters) and the ſum is 5708, which is the ſquare of a mean diameter: which divide by 359.05, and the quotient will be 15.895 ale gallons the area: which multiply by 34 (the depth) and the product will be 540.43 gallons, the content.

By Scale *and* Compaſſes.

Add the two diameters together, and take half the ſum, which is 75.5, which take for a mean diameter (though it is not exact, yet it will be near enough the truth, if the difference between the diameters be not great) extend the compaſſes from 18.95, the gage-point for ale, to 75.5, the mean diameter; that extent will reach from 34, the depth, to a fourth number, and from that to 540.4 gallons, the content.

And if you extend the compaſſes from 17.15, the gage-point for wine, to 75.5, that extent will reach from 34 (twice turned over) to 659.7 gallons of wine.

Laſtly, If you extend the compaſſes from 16.64, the *Iriſh* gage-point, to 75.5, that extent will reach from 34 (twice turned over) to 700.3 *Iriſh* gallons.

The method uſed by the gagers for all ſuch tuns, is to take the diameter in the middle of every 10 inches, that is, at five inches from the bottom, and at 15, and at 25, *&c.*

Then they find the area to every one of theſe diameters, and enter them in their books: then, when they ſurvey, they take the wet inches and parts that the liquor in the tun is in depth, and every 10 inches they take the reſpective areas and remove the ſeparating point one place towards the right hand, and for what odd inches of the depth above the even tens, they multiply the next area by them, and ſo add all the ſeveral products together, and the total will be the gallons of liquor in the tun.

Example. Suppoſe the diameter at 5 inches from the bottom be 64 inches, and at 15 inches from the bottom 67 inches, and at 25 inches, 70 inches, and at 35 inches from the bottom, the diameter is 73 inches. Now the area anſwering 64 inches, is 11.4078 ale gallons, and to 67 inches is 12.5023 gallons; and the area to 70 inches is 13.647 gallons, and to 73 is 14.8418 gallons: then ſuppoſing the depth of the liquor in the ſaid tun be found to be 33.6 inches; now, to caſt up this gage, firſt in the area anſwering to 64 inches, being multiplied by 10, that is, by removing the ſeparating point a place towards the right hand, it will be 114.078 gallons, and the next will be 125.023, and the next 136.47 gallons: now theſe three will be the content to 30 inches deep. Then to find the content of the 3.6 inches, multiply the next area

14.8418 by 36, and the product is 53.4305; add all theſe together, and the ſum is the whole quantity of liquor in the tun.

The content at 10 inches deep	114.078
The content at the next 10 inches	125.023
The content of the next 10 inches	136.470
The content of the 3.6 inches	53.430
The whole quantity of liquor in the tun, ale gallons	429.001

PROBLEM VIII.

To find the drip or fall of a tun.

SUPPOSE the tun laſt mentioned was ſo placed, that when the bottom is but juſt covered on one ſide, the liquor is 4 inches deep on the ſide oppoſite: how much muſt be allowed for the fall of this tun; that is, how much liquor is there in the tun?

The diameter in the middle of 4 inches from the bottom, is 61.6 inches, and the area anſwering thereunto is 10.568; which multiplied by 2 (that is half 4) the product is 21.136 ale gallons; and ſo much liquor will juſt cover the bottom.

But ſuppoſe it was ſet ſo much on one ſide, as to be 30 inches deep on one ſide, when the liquor on the oppoſite ſide juſt cuts between the bottom and ſtaves, how much liquor will there be in the tun?

Square the bottom diameter, and multiply that ſquare by the top diameter, and divide the laſt product by the ſum of the diameters, and to the

Quotient add the ſquare of the bottom diameter, and divide the ſum by 1077.15 for ale, or by 882.36 for wine, or by 831.15 for *Iriſh* Gallons, multiply the quotient by the depth, the product is the content.

The bottom diameter of the forementioned tun is 61 inches, and the diameter at 30 inches from the bottom is 71.5 inches; the ſquare of 61 is 3721, which multiplied by 71.5, the product is 266051.5; this divided by 132.5 the ſum of the diameters, the quotient is 2007.936: to which add 3721, (the ſquare of 61) and the ſum will be 5728.936; this divided by 1077.15, the quotient is 5.3186; which multiplied by 30, the depth, the product is 159.558, the gallons of liquor in the tun.

When the fruſtum of a cone, or pyramid, is cut by a diagonal plane through the extremities of the diameters, as the liquor in the tun repreſents, ſuch ſolid is called a hoof. (*Vide Ward's* Young Mathematician's Guide, page 415.)

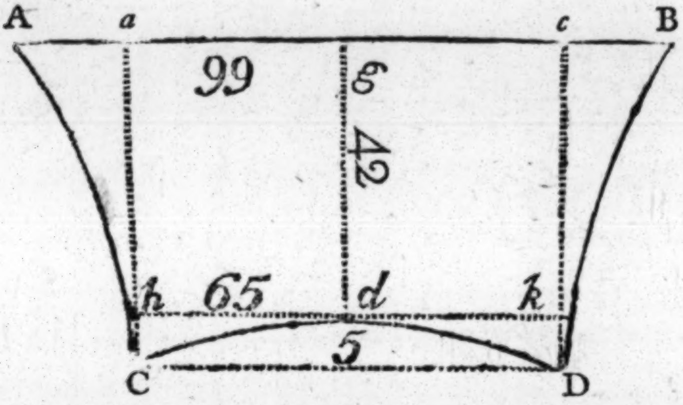
If it be the hoof of a ſquare fruſtum, inſtead of dividing by 1077.15, divide by 846 for ale, or by 693 for wine, or by 652.3 for *Iriſh* gallons. All the reſt of the work is the ſame.

PROBLEM IX.

To Gage a COPPER.

LET ABCD be a ſmall copper to be gaged. Take a ſmall chord, or packthread, make one end faſt at A, and extend the other to the oppoſite ſide of the copper at B, where make it faſt, or cauſe ſome perſon to hold it very ſtraight; then ſet one end of the inſtrument in the bottom of the copper at C, and move it to and fro till you find the neareſt diſtance

tance to the thread (as at *a*) this diſtance, *a* C, is the depth of the copper, which ſuppoſe to be 47 inches.



In like manner ſet the end of the rule upon the top of the crown at *d.* and take the neareſt diſtance to the thread, (as *dg*) which ſuppoſe 42 inches, this ſubtracted from *a* C 47, the remainder 5 is the altitude of the crown.

To find CD, the diameter of the bottom of the crown:

Meaſure AB, the diameter of the top, which admit it be 99 inches, then hold a thread ſo as a plummet at the end thereof may hang juſt over C, by which means you will find the diſtance *a*A. Do the like on the other ſide, ſo will you find alſo the diſtance *c*B, which ſuppoſe 17.5 inches each; add theſe two together, and ſubtract their ſum, *viz.* 35, from 99 and the remainder is 64 inches, the diameter at the bottom of the crown: the diameter which touches the top of the crown may be found by the ſliding rule to be 65 inches.

Now to find the content of the copper from the crown upwards, that is, the part AB*kh*, the depth *gd* being 42 inches, you may take a diameter in the middle of every 6 inches of the depth, which ſuppoſe to be as in the ſecond column of the following table, the numbers in the third column are the reſpective areas in ale gallons, found by *Problem* III. the fourth column ſhews the content of every 6 inches; all which being added together, the ſum will be the content of that part AB*kh*; that is, ſo much as it will hold after the crown is covered.

Now, if the crown be taken for the fruſtum of a ſphere, the content (by the latter part of Sect. XI. *Page* 187) will be found to be 28.75 gallons.

But may be more readily found, very near the truth thus:

The diameter CD was found to be 64, and the area to this diameter is 11.408; this multiplied by half the crown's altitude, *viz.* by 2.5, gives 28.52 ale gallons, the content of the crown.

The content of the part *hk*DC is 57.935 ale gallons, from which ſubtract the content of the crown, 28.52, and the remainder is 29.415 ale gallons, and ſo much liquor will juſt cover the crown.

Parts of the depth	Diameters	Areas	Content of every ſix inches
6	95.3	25.2945	151.767
6	90.1	22.6095	135.657
6	85.0	20.1223	120.734
6	83.	17.8246	106.947
6	75.2	15.7499	94.499
6	70.5	13.8426	83.056
6	66.	12.1319	72.791
The ſum			705.451
To juſt cover the crown			29.415
The whole content in ale gallons			794.866

By Scale *and* Compaſſes.

You may find the areas anſwering to every one the diameters thus:

Extend the compaſſes from the gage-point to the diameter, that extent, being twice turned over from 1, will at laſt fall upon the area of that circle; or being twice turned over from 6, will give the content of that 6 inches of the depth.

Example. Extend the compaſſes from 18.95 (the gage-point) to 95.3 that extent, turned twice over from 6, will at laſt fall upon 151.76 gallons the content of the firſt 6 inches. And ſo of the reſt.

PROBLEM X.

To compute the content of any cloſe Caſk.

IN order to perform this difficult part of gaging, the three following dimenſions of the caſk muſt be truly taken,

Viz. { The bung-diameter,
The head-diameter,
The length of the caſk, } within the caſk.

In taking theſe dimenſions it muſt be carefully obſerved,

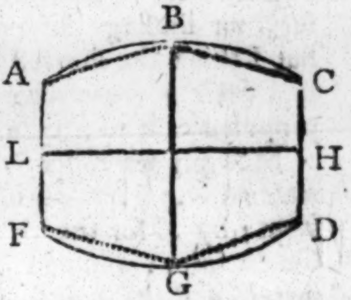
1. That the bung-hole be in the middle of the caſk; alſo, that the bung-ſtaff, and the ſtaff oppoſite to the bung-hole, are both regular and even within.

2. That the heads of the caſk are equal, and truly circular; if ſo, the diſtance between the inſide of the chine to the outſide of its oppoſite ſtaff, will be the head diameter within the caſk, very near.

3. With a ſliding pair of *callipers*, made for that uſe, take the ſhorteſt diſtance, or length, between the outſides of the two heads; from that length ſubtract $1\frac{1}{2}$ inch, more or leſs, according to the largeneſs of the caſk, for the thickneſs of the head; the remainder will be the length of the caſk within. But if the caſk be empty, you may take the length by putting a ſtraight rod in at the tap-hole and allow for the thickneſs of the head.

Now, by theſe dimenſions, one would think the content of the caſk was perfectly limited; but it will be eaſy to perceive, by the following figure, that the diameters and length of one caſk may be equal to thoſe of another, and yet one of thoſe caſks may contain ſeveral gallons more than the other.

As for inſtance, the figure ABCDF is ſuppoſed to repreſent a caſk: then it is plain, that if the outward curved lines, ABC and FGD, are the bounds, or ſtaves of the caſk, it muſt needs hold more than if the inner pricked lines were the bounds of ſtaves; and yet the bung diameter BG and the head-diameter CD and AF, and the length LH, are the ſame in both thoſe caſks.



Whence it appears that no one general rule can be given, whereby the content of all ſorts of caſks can be gaged. And therefore Gagers do uſually ſuppoſe every caſk to be in ſome of theſe forms.

1. The middle fruſtum of a ſpheroid.
2. The middle fruſtum of a parabolic ſpindle.
3. The lower fruſtums of two equal parabolic conoids.
4. The lower fruſtums of two equal cones.

1. If the ſtaves of the caſk be very much curved (as the outward lines of the laſt figure) then the caſk is ſuppoſed to be the middle fruſtum of a ſpheroid.

2. If the ſtaves between the bung and the head be ſomething leſs curved, then the caſk is taken to be the middle fruſtum of a parabolic ſpindle.

3. If the ſtaves (between the bung and the head) be very little curved, then the caſk is taken to be the lower fruſtums of two equal parabolic conoids abutting or joining together, upon one common baſe.

4. If the ſtaves (between the bung and the head) be ſtraight, as the pricked line in the laſt figure, then the caſk is taken to be the lower fruſtums of two

equal cones, abutting or joining together upon one common baſe.

There are ſeveral rules laid down in books of gaging, for finding the content of each ſeveral form, but I think the ſhorteſt and moſt practical way is to find ſuch a mean diameter, which will reduce the propoſed caſk to a cylinder; thus,

Multiply the difference of the bung and head diameters by .7 for the ſpheroid ; by .65 for the ſecond form ; by .6 for the third form, and by .55 for the fourth form ; and add the product to the head diameter, and the ſum is a mean diameter.

Example. Suppoſe the bung-diameter be 32 inches, the head diameter 24 inches, and the length 40 inches, the content in each variety is required.

The difference between the bung and head diameters is 8, which multiplied by 7, the product is 5.6 ; which added to the head-diameter, the ſum is 29.6 for the mean diameter ; the area anſwering thereunto will be found by *Prob.* III. to be 2.44 ale gallons : which multiplied by the length, the product is 97.4 gallons, and ſo much is the content, if it be of the firſt form.

Again, If the difference of the diameters 8, be multiplied by .65, the product will be 5.2 ; which added to the head diameter, the ſum is 29.2 for the mean diameter ; and the area anſwering thereunto is 2.3746 gallons ; which multiplied by 40, the length, the product is 94.98 gallons, the content, if it be of the ſecond form.

Again, If the difference 8, be multiplied by 6, the product is 4 8 ; which added to the head diameter, the ſum is 28.8, the mean diameter ; the area thereunto is 2.31 gallons ; which multiplied by 40, gives the content 92.4 gallons for the third form.

Again, the difference 8, multiplied by 55, the product is 4.4 ; which added to the head diameter, makes the mean diameter 28.4 ; the area thereof is 2.2463, 2.2463, which multiplied by 40, the product is 89.85 gallons for the third form.

By Scale *and* Compaſſes.

Extend the compaſſes from the gage-point 18.95, to the firſt mean diameter 29.6, that extent will reach from the length 40, to a fourth number, and then to the content 97.4 ale gallons.

Again, Extend from 18.95 to 29.2, (the ſecond mean diameter) that extent, turned twice over from 40, will at laſt fall upon 94.48 gallons.

Again, Extend from 18.95 to 28.8, (the third mean diameter) that extent, turned twice over from 40, will at laſt fall upon 92.4 gallons.

Again, Extend from 18.95 to 28.4, (the fourth mean diameter) that extent, turned twice over from 40, will at laſt fall upon 89.85 gallons.

Although I have all along made uſe of the line of numbers upon the common two foot or eighteen inch rules, for the reaſons mentioned in the *Preface*, yet the rules may eaſily be applied to the ſliding rule, thus: to find the area of a circle in gallons, ſet the gage-point upon D, (that is, a ſingle line of numbers) to 1 upon C, (that is, a double line) then againſt any diameter upon D, is the area upon C, thus:

To find the content of the caſk laſt mentioned. The firſt form:

Set the gage-point 18.95 upon D, to the length 40 upon C; then, againſt the mean diameter 29.6 upon D, is 97.4 gallons, the content upon C.

And againſt 29.2 (the next mean diameter) on D, is 94.98 gallons on C.

And againſt 28.8 (the next mean diameter) on D, is 92.4 gallons on C.

And againſt 28.4 (the laſt mean diameter) on D, is 89.85 gallons on C.

All done without removing the ſlider.

A TABLE *of the Segments of a Circle whoſe Area is Unity.*

V. S.	Segm	V. S.	Segm	V. S.	Segm	V. S.	Segm
1	.0017	99	·9983	26	.2066	74	·7924
2	.0048	98	·9952	27	.2178	73	·7832
3	.0087	97	·9913	28	.2292	72	7708
4	.0134	96	.9866	29	.2407	71	·7593
5	.0187	95	·9813	30	·2523	70	·7477
6	.0245	94	·9755	31	.2640	69	·7360
7	.0308	93	·9692	32	·2750	68	·7241
8	.0375	92	9625	33	·2878	67	·7122
9	·0446	91	·9554	34	·2998	66	.7002
10	.0520	90	·9480	35	·3119	65	.6881
11	·0598	89	·9402	36	·3241	64	.6759
12	.0680	88	·9320	37	·3364	63	.6636
13	.0764	87	9236	38	·3487	62	.6513
14	.0851	86	·9149	39	·3611	61	.6389
15	·0941	85	·9059	40	·3735	60	.6265
16	·1033	84	.8976	41	·3860	59	.6140
17	·1127	83	.8873	42	·3986	58	.6014
18	·1224	82	.8776	43	·4112	57	·5888
19	·13[illegible]3	81	8677	44	·4238	56	·5762
20	·1424	80	8576	45	·4364	55	·5636
21	·1526	79	·8474	46	·4491	54	·5509
22	.1631	78	.8369	47	·4618	53	·5382
23	·1737	77	.8263	48	·4745	52	5255
24	·1845	76	·8155	49	·4873	51	·5127
25	·1955	75	8045	50	·5000	50	.5000

The Uſe of the Table of Segments.

§ 1. *Is to find the Ullage, or Quantity of Liquor remaining in a Caſk, whoſe Axis is parallel to the Horizon, the ſurface of the Liquor cutting the Heads of the Caſk.*

The RULE *is,*

TO the wet or dry inches of the bung diameter add a competent number of ciphers, then divide

vide it by the whole diameter, the quotient found in the table under the title V. S. gives a ſegment; which multiplied by the whole content of the caſk, the product ſhews the quantity of liquor in the caſk if the dividend was the wet inches, or the ullage, if it was the dry.

Let there be a caſk in form of a cylinder, whoſe bung diameter is 29 inches, the dry part 13, and the wet 16, and the content 80 gallons; how many gallons are wanting to fill the caſk?

Divide the dry inches 13, by 29 the bung diameter, and the quotient is 448; find the two firſt figures .44 under V. S. and the ſegment againſt it is .4238, to which add a proportional part for the 8. and the whole ſegment will be .4333; which multiplied by the content of the caſk, the product will be 34.664 gallons; and ſo much the caſk wants of being full.

NOTE, If the caſk be in the form of a cylinder or near that figure, the table will give the ullage exact enough; but if it be a ſpheroid caſk, then uſe the following method:

1. By the bung and head diameters, find ſuch a mean diameter as you judge will reduce the propoſed caſk to a cylinder, and then find its content.
2. From the bung diameter ſubtract the mean diameter, and take half the difference.
3. From the wet inches ſubtract the ſaid half difference; reſerve this difference, then uſe this proportion;

As the mean diameter: is to 100,
(the diameter of the tabular circle) : :
ſo is the reſerved difference :
to a verſed ſine in the table.

Then, if the tabular ſegment be multiplied into the content (as before) the product will be the quantity of liquor in the caſk.

Example. Let the caſk be the ſame as in page 317 of the firſt form, where the bung diameter is 42 inches, and the mean diameter 29.6, and the content 97.4 gallons; and ſuppoſe the wet inches 19. to find the quantity of liquor in the caſk.

From	32	From	19
Subtr.	29.6	Subtr.	1.2
Rem.	2.4	Rem.	17.8 reſerved.
Half.	1.2		

As 29.6 : 100 : : 17.8 : 60, the V. S.

The ſegment to 60 is .6265, which multiplied by 97.4, the content, the product is 61 gallons, the quantity of liquor in the caſk.

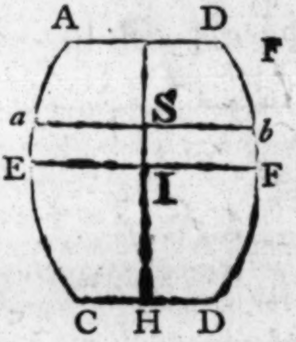
If the dry inches had been given, by the ſame method you might have found the ullage, or what the caſk wanted of being full.

§ 2. *To find what quantity of liquor is in a Caſk, when its Axis is perpendicular to the Horizon,* viz. *when it ſtands upright upon one of its Heads.*

TO do this, you muſt know how to calculate the area of any circle, between the bung and head, whoſe diſtance from the bung, or middle of the caſk is given; which may be done by this proportion:

As the ſquare of half the length of the caſk : is to the difference between the bung and head areas : : ſo is the ſquare of any circle's diſtance from the bung : to the difference between the bung area and the area of that circle, *viz.* the area of the liquor's ſurface.

Then, from the bung-area ſubtract one third part of the aforeſaid difference, *viz* between the bung-area and the area of the liquor's ſurface; multiply the remainder by the liquor's diſtance from the bung, and the product will ſhew what quantity of liquor is either above or under half the content of the caſk.



Example. Let us again ſuppoſe the caſk in page 317 whoſe length is 40 inches, bung-diameter 32, and head-diameter 24, and ſuppoſe the wet inches SH 26 inches.

The ſquare of half the length is 400, the diſtance of the liquor's ſurface from the bung SI is 6, whoſe ſquare is 36; the area of the bung-diameter 2.8519 ale gallons, and the area of the head-diameter 1.6042: the difference 1.2477. Then,

As 400 : 1.2477 : : 36 : .0751.

One third is = .0250

from 2.8519 Bung area,
Subtr. .0250 a third of the difference.

Rem. 2.8269

6 Multip. diſtance from the bung.

16.9614 Content above the bung.
And 48.7 Half the content of the caſk.

65.66 The quantity of liquor in the caſk.

PROB-

PROBLEM XI.

Gaging of MALT.

TO find the quantity of malt in a ciſtern, or upon a floor.

Firſt, find the area of the baſe in buſhels, by multiplying the length by the breadth, and dividing the product by 2150.42, or only by 2150, and multiply that area by the mean depth : (how to take the mean depth, ſee *Problem* II.) If the baſe be circular, or oval, divide by 2738. (See *Problem* I.)

Example. There is a ciſtern whoſe length is 84 inches, and breadth 54 inches, and the mean depth is 43.6 inches ; what is the content ?

Multiply 84 by 54, and the product is 4536; which divide by 2150, and the quotient is 2.1097 buſhels, the area of the bottom at one inch deep ; which multiplied by the depth 43.6, and the product is 91.98 buſhels, the content.

Example. Suppoſe a quantity of malt upon a floor whoſe length is 245 inches and the breadth 184 inches, and the mean depth 5.6 inches, how many buſhels are there ?

Multiply 245 by 184, and the product is 45080 which divided by 2150, the quotient is 20.967, the area of the baſe, which multiplied by the mean depth, the product is 117.4 buſhels, the content.

By the Sliding Rule

There is an inverted line of numbers upon ſome ſliding rules marked with the letter M, which was contrived purpoſely for gaging of malt ; and there is a double line of numbers upon the rule, and upon the ſlider two double lines of numbers ; all theſe are of equal radius, and all work together at once : thus ſet the length and breadth againſt one another upon the inverted line, and that which ſlides by it ; then, on the other edge of the rule, againſt the depth, you will find the content in buſhels: thus, in the firſt *Example*, ſet 54 upon the ſlider againſt 84 upon the inverted line, and then againſt 43.6, upon the other part of the rule, is 91.98 upon the ſlider.

Again, in the ſecond *Example*, ſet 184 upon the ſlider to 245 upon the inverted-line, and againſt 5.6 upon the other part of the rule is 117.4 upon the ſlider.

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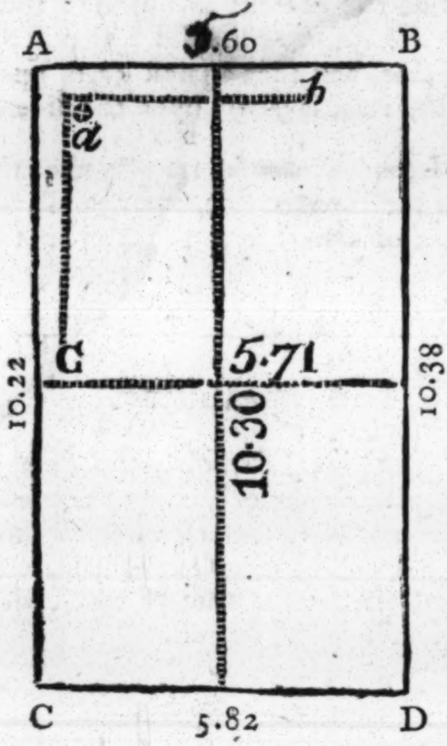
§ II. *Of* Land-meaſuring.

I SHALL not here give the whole art of *Surveying*, but ſuch practical rules only as may be uſeful to the country Graziers and Farmers, whereby they may find the true content of any piece of land, and that by the chain only; and for want of that, with a pole or ſtick of half a rod in length.

PROBLEM I.

To find the Content of a Piece of Land, in the Form of a right-angled Parallelogram or Long Square, or what is ſomething near that Form.

TO know whether any angle in a field be a right-angle or not, you may take a piece of board about 4 or 5 inches broad, and an inch thick, either round or ſquare, and with a ſaw cut two kerfs, croſſing each other at right angles, and bore a hole in the middle of the back ſide, to put it upon the end of a ſtick; this will repreſent the inſtrument called a croſs.



Suppoſe you would obſerve the angle A, to know whether it be a right angle, (or near thereunto,) prick up your ſtick with the croſs upon it, a little diſtance from the fence, as at *a*, and having ſet up two marks, as at *b* and *c*, of equal diſtance from the fence, turn one of the ſlits directed towards *b*; and then, if the other be directly pointing to *c*, it is a right-angle.

To meaſure ſuch a piece of ground as this figure above: if you meaſure round, and add the oppoſite ſides together, and take half the ſum (if they be not equal) or elſe meaſure down about the middle of the length and middle of the breadth, thus the ſide AB being meaſured, it will be 5.60, that is 5 chains and 60 links; and the oppoſite ſide CD is 5 chains 82 links; the half ſum thereof is 5.71: and the ſide BD is 10.38, and the ſide AC 10.22; and the half ſum thereof 10.30: (it will be the ſame thing if you meaſure about the middle of the breadth) then multiply this mean length and mean breadth together, *viz.* 10.30, by 5.71, and the product is 5[illegible]8130; which divide by 10, becauſe 10 ſquare chains is an acre, by removing the ſeparating point one place towards the left hand, and it will be 5.88130, that is, 5 acres and 88130 parts; which multiplied by 4, and prick

58·8130

prick off 5 places, and it will be 3.52520; which 3 towards the left hand are 3 roods; then multiply the decimal parts by 40, and prick off 5 places, and it will be 21.00800; which 21 towards the left hand are 21 perches.

A. R. P.
So the whole content is — 5 3 21

See the Work.

5.71
10.30

17130
571

5.88130
4

3.52520
40

21.00800

A. R. P.
5 : 3 : 21

NOTE, The chain here made uſe of is 4 poles, or rods, in length; the whole chain being 100 links.

But becauſe every man that may have occaſion to meaſure a piece of land can't procure a chain, I will therefore ſhew how you may meaſure a piece of land only with a ſtick of half a rod in length, that is, 8 feet and 3 inches; but in *Ireland* 'tis 10 feet 6 inches; which ſtick divided into five equal parts, ſo will the whole rod be divide into ten parts, and will thereby be adapted to decimal arithmetic.

But becauſe each of thoſe parts of the ſtick are ſomething large, (each part being 19 inches and 8 tenths in *Great Britain*, but 25.2 inches in *Ireland)* it will be neceſſary to take your dimenſions to half of one of thoſe parts, and then, for that half part, ſet 5 in the place of ſeconds thus; ſuppoſe 3 parts and a half, ſet it down thus, .35.

PROBLEM II.

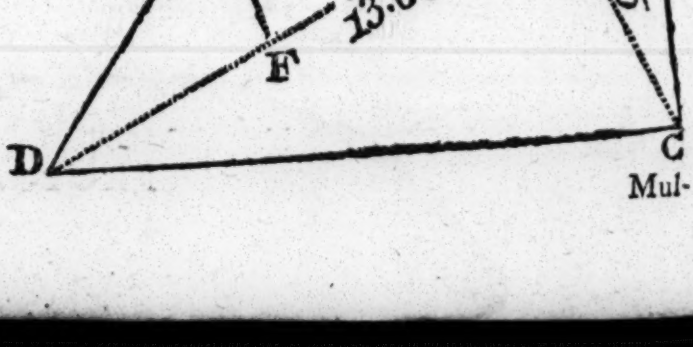
LET us ſuppoſe a field in the form of a long ſquare, whoſe length is 45 rods, 5 parts and a half, and the breadth 31 rods, 4 parts and a half; what is the content?

Multiply the length and breadth together, and divide the product by 160, (becauſe 160 ſquare rods are in an acre) and the quotient is acres.

			A.	R.	P.
45.55	16|0)143|2(8				
31.45	128	*Facit*	8	3	32
22775	4|0)15|2(3				
18220	12				
4555	—				
13665	32				
1432.5475					

PROBLEM III.

SUPPOSE a piece of ground in the form of a trapezium, the diagonal BD 13 chains 60 links, the perpendicular CE 6 chains 25 links, and the perpendicular AF 3 chains 42 links, what is the content?



Multiply the diagonal by half the ſum of the perpendiculars: *See* Sect. VI. *of* Chap. I. Part II.

CE=6.25	13.60=BD	
AF=3.42	4.83	
Sum 9.67	4080	A. R. P.
	10880	*Facit* 6 2 11
Half 4.83	5440	
	6.56880	
	4	
	2.27520	
	40	
	11.00800	

By Rods, thus,

CE=25. Rods.	19.34
AF=13.68	54.4=BD
Sum 38.68	7736
	7736
Half 19.34	9670
	16|0)105|2.096(6
	96
	4|0)9|2(2
	8
	12

A. R. P.
Facit 6 2 12

To take the dimenſions of the field.

Begin at the angle B, and meaſure in a direct line towards D; but when you come at E ſet up your croſs, and direct one of the ſlits to D, and then look through the other ſlit, and if it exactly hits the angle C, then you are juſt in the place where the perpendicular will fall; but if it does not exactly hit the point, move backwards or forwards till it does ſo; then meaſure the perpendicular, and ſet down the chains and links, or the rods and parts; then continue your meaſure toward D; but when you come to F, ſet up your croſs, and try (as above directed) whether you be in the place where the perpendicular will fall: then meaſure the perpendicular AF, and ſet down the chains and links, or rods and parts; then continue your meaſure to D, and ſet down the meaſure of the whole diagonal. This way of meaſuring is very exact and true; but the common way uſed by the graziers and farmers, is to meaſure round the field, and to take half the ſum of the oppoſite ſides for a mean ſide; but the laſt mentioned piece of ground being meaſured ſo will come to

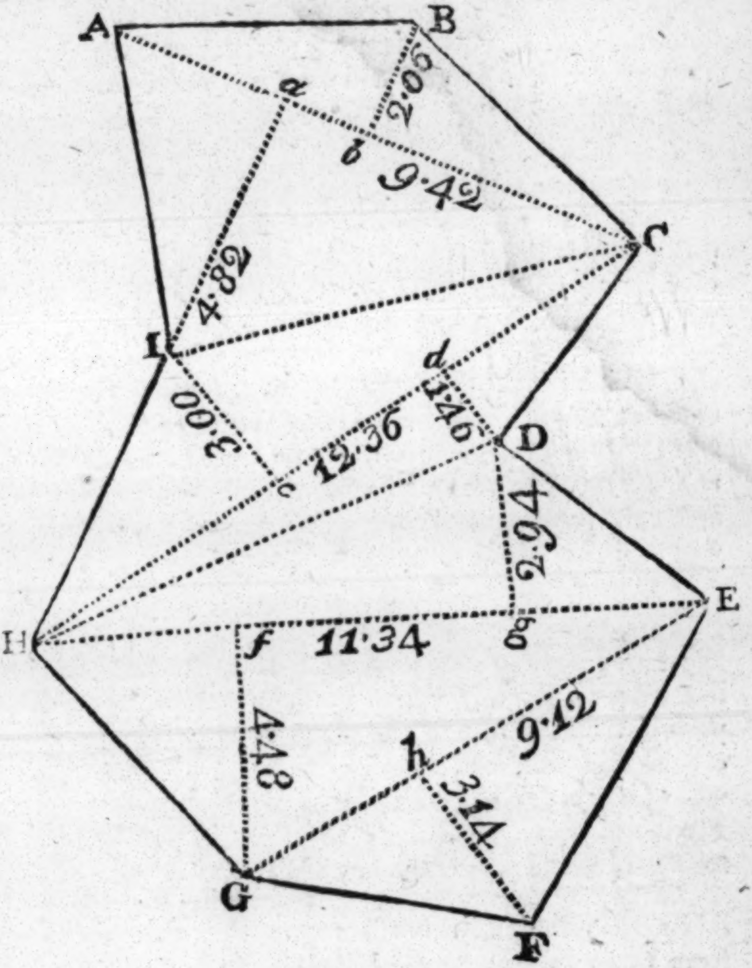
A.	R.	P.		R.	P.	
7	0	22,	which is	2	10	more than the truth.

PROBLEM IV.

How to meaſure an irregular field.

THE way to meaſure irregular land, is to divide it into trapeziums, and triangles; thus,

Firſt, view over the field, and ſet up marks at every angle, and by thoſe marks you may ſee where to have a trapezium, as ABCI in the following figure. Then



Then begin and meaſure in a direct line from A towards C: but when you come to *a* ſet up your croſs, and try whether you be in a ſquare to I, (as is before directed) and then meaſure the perpendicular *a* I, which is 4.82; then meaſure forward again towards C: but when you come to *b*, ſet up your croſs, and try whether you be in the place where the

* Ch. L.

the perpendicular will fall; then meaſure the perpendicular *b*B, which is $\overset{\text{Ch. L.}}{2.06}$; then continue your meaſure to C, and you will find the whole diagonal 9Ch. 42L.

Then proceed to meaſure the trapezium CDHI, beginning at C, and meaſuring along the diagonal line towards H; but when you come at *d* ſet up your croſs, and try if you be right in the place where the perpendicular will fall: meaſure the perpendicular *d*D, which is $\overset{\text{Ch. L.}}{1.46}$, and then meaſure forward till you come at *c*, and there, with your croſs, try if you be right in the place where the perpendicular will fall, and meaſure the perpendicular *c*I, which is 3 chains; and from *c* continue your meaſure to H, and you will find the whole diagonal 12Ch. 36L.

Then proceed to meaſure the trapezium HGED, beginning at H, and meaſuring along the diagonal line towards E, but when you come to F, try with your croſs if you be in the place where the perpendicular will fall; and meaſure the perpendicular FG, which is 4.48, then continue on your meaſure from F till you come to G, and there try if you be in a ſquare with the perpendicular GD; and meaſure the ſaid perpendicular, which is $\overset{\text{Ch. L.}}{2.94}$; then meaſure on from G to E, and you will find the whole diagonal to be $\overset{\text{Ch. L.}}{11.34}$.

Then meaſure the triangle EFG, beginning at E, and meaſure along the baſe EG till you come at *h*, and there, with your croſs, try if you be in the place where the perpendicular will fall; and meaſure the perpendicular *h*F, which is $\overset{\text{Ch. L.}}{3.14}$; continue your meaſure to G, and you will find the whole baſe to be 9 Ch. 12L. : ſo have you finiſhed the whole field.

I have been the larger upon the explanation of this *Problem*, becauſe moſt grounds lie in ſuch irregular forms.

Caſt up the three trapeziums ſeverally, and alſo the triangle, and add all the ſeveral areas together into one ſum which will be the area of the whole irregular plot.

See the Work.

*b*B=2.06	9 42	See Sect. VI. Chap. I. Part II.
*a*I=4.82	3.44	
Sum 6.88	3768	
	3768	
Half 3.44	2826	
	3.24048=Area of ABCI.	

*d*D=1.46	12.36
c I =3.00	2.23
Sum 4.46	3708
	2472
Half 2.23	2472
	2.75628=Area of CIHD.

FG=4 48	11.34
GD=2.94	3.71
Sum 7.42	1134
	7938
Half 3.71	3402
	4.20714=Area of HGED.

Baſe = 9.12

Half = 4.56 See Sect. V. Chap. I.
Perpen. 3.14 Part II.

1824
456
1368

1.43184 = Area of the triangle EFG.
3.24048 = Area of ABCI.
2.75628 = Area of CIHD.
4.20714 = Area of HGED.

Sum 11.63574 = Area of the whole.
4

2.54296
40

21.71840

A. R. P.
Facit 11 2 21

FINIS.

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